Lesson 13: Application Problems with Quadratic Equations

Lesson Objectives:

- Student will solve quadratics by using the quadratic formula.
- Student will apply methods to solve quadratic equations used in real world situations.

Quadratic Word Problems

Short videos: **Projectile Word Problem**

Time and Vertical Height with Graphing Calc

Area Word Problem

Motion Word Problem

Business Word Problem

Skid Mark Problem

Geometry Word Problem

Types of Quadratic Applications

I. Projectile motion

A "projectile" is any object that is thrown, shot, or dropped. Usually the object is moving straight up or straight down.

Typical questions are:

1. What is the height (above ground level) when the object is launched?
2. How long before the object hits the ground after launch?
3. What is the maximum height of the object?

4. After how many seconds is the object at its maximum height?

Ex. 1 An object is launched at 19.6 meters per second (m/s) from a 58.8-meter tall platform. The equation for the object's height s at time t seconds after launch is $s(t) = -4.9t^2 + 19.6t + 58.8$, where s is in meters.

1. What is the height above the ground when the object is launched?
2. How long before the object hits the ground after launch?
3. What is the maximum height of the object?
4. When does the object reach its maximum height?

Solution:

1. $S(t) = -4.9(0)^2 + 19.6(0) + 58.8$

Answer: Height = 58.8 m

b. $S(t) = 0$ when the object hits the ground

$-4.9(t^2 - 4t - 12) = 0$
$(t - 6)(t + 2) = 0$

$t = 6$ and $t = -2$

Answer: The object strikes the ground six seconds after launch.

C&d. The maximum height of the object and time when it reaches its maximum are located at the vertex of the parabola.

Vertex: x-coord = $-19.6/(2*-4.9) = 2$ sec

y-coord = $-4.9(2)^2 + 19.6(2) + 58.8 = 78.4$ m

Answer: The maximum height of the object is 78.4 m

Answer: The object reaches its maximum height after 2 sec

Note: The leading coefficient **is** negative, since gravity pulls downward, and will either be "4.9" (if units are "meters") or "16" (if units are "feet"). In general, the format is:

$$\boldsymbol{s(t) = -gt^2 + v_0t + h_0}$$

...where "g" is the "4.9" or the "16" from gravity, "v_0" is the initial velocity, and "h_0" is the initial height. Memorize this equation.

Ex. 2 An object is launched directly upward at 64 feet per second (ft/s) from a platform 80 feet high. What will be the object's maximum height? When will it attain this height?

Solution*:* $s(t) = -16t^2 + 64t + 80$

$h = {}^{-b}/_{2a} = -(64)/2(-16) = -64/-32 = 2$
$t = s(2) = -16(2)^2 + 64(2) + 80 = -16(4)+128+80 = -64 + 208 = 144$

Answer: It takes two seconds to reach the maximum height of 144 feet.

Ex 3 An object is launched from ground level directly upward at 39.2 m/s. For how long is the object at or above a height of 34.3 meters?

Solution*:* $s(t) = -4.9t^2 + 39.2t$

$34.3 = -4.9t^2 + 39.2t$
$-4.9(t^2 - 8t + 7) = 0$
$(t - 7)(t - 1) = 0$

t = 1 and 7 seconds

Answer: The object is at or above 34.3 meters for six seconds.

II. General problems

Ex. 4 A picture has a height that is 4/3 its width. It is to be enlarged to have an area of 192 square inches. What will be the dimensions of the enlargement?

Solution:

A = lw

Since the height is the length, then A = hw where h = (4/3)w.

$(4/3)w^2 = 192$
$w^2 = 144$
$w = \pm 12$

Answer: The enlargement will be 12 inches by 16 inches.

Ex. 5 The product of two consecutive negative integers is 1122. What are the numbers?

Solution:

n = 1st negative integer

n + 1 = 2nd consecutive integer

$n(n + 1) = 1122$
$n^2 + n = 1122$
$n^2 + n - 1122 = 0$
$(n + 34)(n - 33) = 0$

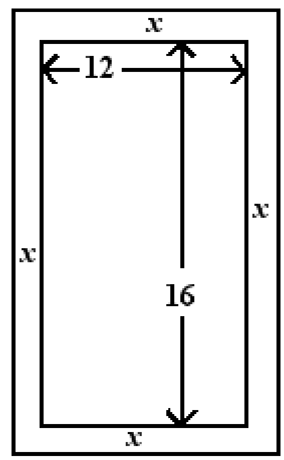
n = -34, 33

n+1 = -33, 34

Answer: The two negative numbers are −33 and −34.

(Note: If the integers were consecutive odd or consecutive even, you would use n, n+2, n+4, etc. to set up the problem.)

Ex. 6 A garden measuring 12 meters by 16 meters is to have a pedestrian pathway installed all around it, increasing the total area to 285 square meters. What will be the width of the pathway?



Solution:
Draw a picture. Label the width of the sidewalk as "*x*".

The total width will be $12 + x + x = 12 + 2x$. The total length will be $16 + x + x = 16 + 2x$.

The new area is given by:

$$(12+2x)(16+2x) = 285$$

$$192 + 56x + 4x^2 = 285$$
$$4x^2 + 56x - 93 = 0$$

$$x = \frac{-(56) \pm \sqrt{(56)^2 - 4(4)(-93)}}{2(4)}$$
$$= \frac{-56 \pm \sqrt{3136 + 1488}}{8}$$
$$= \frac{-56 \pm 68}{8} = -15.5, 1.5$$

Answer: The width of the pathway will be 1.5 meters.

III. Max/min problems

Ex. 7 You have a 500-foot roll of fencing and a large field. You want to construct a rectangular playground area. What are the dimensions of the largest such yard? What is the largest area?

$A = Lw = (250 - w)w = 250w - w^2 = -w^2 + 250w$

$h = {}^{-b}/_{2a} = -(250)/2(-1) = -250/-2 = 125$

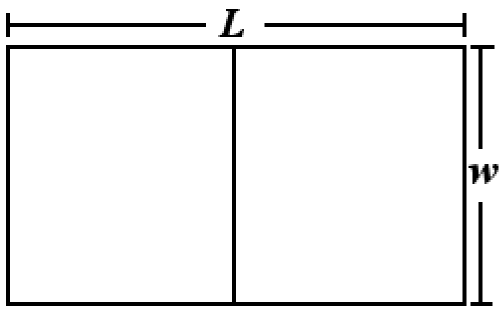
$k = A(125) = -(125)^2 + 250(125) = -15\,625 + 31\,250 = 15\,625$

Answer: The largest area will have dimensions of 125' by 125', for a total area of 15 625 square feet.

Ex. 8 You have a 1200-foot roll of fencing and a large field. You want to make two paddocks by splitting a rectangular enclosure in half. What are the dimensions of the largest such enclosure?

Solution

The diagram:



The "perimeter" is $2L + 3w = 1200$.

$L + 1.5w = 600$
$L = -1.5w + 600$

$A = Lw = (-1.5w + 600)w = -1.5w^2 + 600w$

$h = {}^{-b}/_{2a} = -(600)/2(-1.5) = -600/-3 = 200$

Answer: The paddock should be 300' by 200', with the divider running parallel to the 200-foot-long side.

Ex. 9 Your factory produces lemon-scented widgets. You know that each unit is cheaper, the more you produce, but you also know that costs will eventually go up if you make too many widgets, due to storage requirements. The guy in accounting says that your cost for producing *x* thousands of units a day can be approximated by the formula $C = 0.04x^2 - 8.504x + 25302$. Find the daily production level that will minimize your costs.

Solution:

$h = {}^{-b}/_{2a} = -(-8.504)/2(0.04) = 8.504/0.08 = 106.3$

Answer: Minimizing costs happen when the factory produces 106 300 units a day.

Grading Rubric:

Your grade will be calculated by the sum of the points earned for each question. Points are earned according to the chart below.

To get a 10: A total score of 10 on your first submission, or within the first revision.

To get a 9: A total score of 9 or more after your first revision.

To get an 8: A total score of 8 or more after your first revision.

To get a 7: A total score of 7 or more after your first revision.

To get a 6: A total score of 6 or more after your first revision.

To get a 5: Any score lower than a 6; Plagiarism – purposeful or mistaken, which will lower your final grade for the course (So, be very careful when posting your work!); lack of effort, disrespect, or attitude. Lesson requirements have not been met.

Questions 1-5 (Total points possible = 10)	Answer is correct with method indicated in work. 2 points each	Answer is correct. Work is not clear or is incomplete. 1 point each	Answer is incorrect and work is not given. 0 points each

Assignment:

Solve the problems below. Write the problem, your work, and the solution in the text box below to submit your work. Be sure to show all of your work. Here is a link explaining how to show your work. We suggest saving your work in a word processor.

Solve each problem below showing the steps as indicated in the lesson.

1. A rectangle and a square have the same area. The length of the rectangle is seventy feet more than two times its width. The length of a side of the square is thirty feet. What equation would help you solve for the dimensions of the rectangle? What are the dimensions of the rectangle?

2. Four times the sum of three consecutive odd integers is seven hundred sixty-five less than three times the product of the larger two numbers. What are the three integers?

3. A ball is launched upward at 20 meters per second (m/s) from a 60 meter tall platform. The equation for the object's height s at time t seconds after launch is $s(t) = -4.9t^2 + 20t + 60$, where s is in meters. What is the height above the ground when the object is launched?

4. A ball is launched upward at 20 meters per second (m/s) from a 60 meter tall platform. The equation for the object's height s at time t seconds after launch is $s(t) = -4.9t^2 + 20t + 60$, where s is in meters. How long before the object hits the ground after launch?

5. A ball is launched upward at 20 meters per second (m/s) from a 60 meter tall platform. The equation for the object's height s at time t seconds after launch is $s(t) = -4.9t^2 + 20t + 60$, where s is in meters. What is the maximum height of the object?