Activity 3.1.1 Sum of Interior Angles of Triangles

Triangle Angle Sum Conjecture: The sum of the interior angle measures in every triangle is ____?

On a sheet of blank paper, draw any triangle you like using your ruler. Label the vertices *A*, *B*, and *C*. Measure the three angles of your triangle. Then measure the three angles of your partner's triangle. Record your answers in the appropriate space below.

Your triangle:

Angle measure at vertex *A*: ______

Angle measure at vertex *B*: ______

Angle measure at vertex *C*: ______

Sum of the three angle measures: ________

Your partner's triangle:

Angle measure at vertex *A*: ______

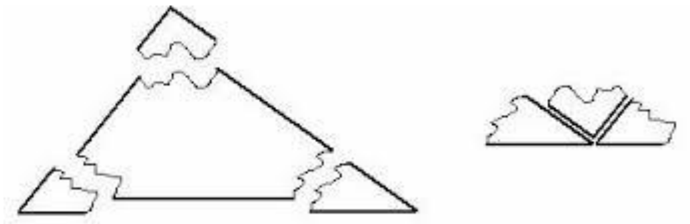
Angle measure at vertex *B*: ______

Angle measure at vertex *C*: ______

Sum of the three angle measures: ________

What conclusions can you make? Do you think this works for all triangles? Discuss this with your partner and write your combined thoughts in the space provided.

Now take the triangle that you drew and cut it out. Tear off the three corners. Rearrange the pieces to form a line. Does this confirm your conjecture or conclusions from the previous exercise? Are you sure? How can you be sure you have created a line?



Review your thoughts on the triangle angle sum conjecture:

Triangle Angle Sum Conjecture: The sum of the interior angle measures in every triangle is ____?

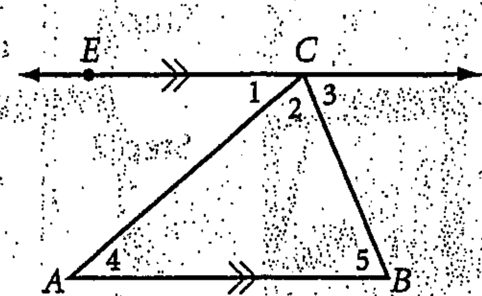
Steps 1 through 5 may have convinced you that the Triangle Sum Conjecture is true, but a proof will explain *why* it is true for every triangle.

ep 6 | Copy and complete the paragraph proof below to explain the connection between the Parallel Lines Conjecture and the Triangle Sum Conjecture.

Paragraph Proof: The Triangle Sum Conjecture

To prove the Triangle Sum Conjecture, you need to show that the angle measures in a triangle add up to _?_. Start by drawing any $\triangle ABC$ and $\overleftrightarrow{EC}$ parallel to side $\overline{AB}$.

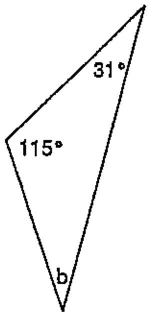
$\overleftrightarrow{EC}$ is called an **auxiliary line**, because it is an extra line that helps with the proof.



In the figure, $m\angle 1 + m\angle 2 + m\angle 3 = 180°$ if you consider $\angle 1 + \angle 2$ as one angle whose measure is $m\angle 1 + m\angle 2$, because _?_. You also know that $\overline{EC} \parallel \overline{AB}$, so $m\angle 1 = m\angle 4$ and $m\angle 3 = m\angle 5$, because _?_. So, by substituting for $m\angle 1$ and $m\angle 3$ in the first equation, you get _?_. Therefore, the measures of the angles in a triangle add up to _?_. ■

Find the measure of angle b.

1)



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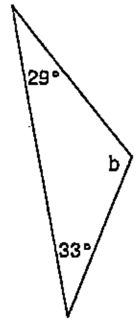

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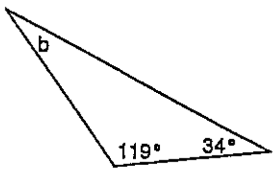

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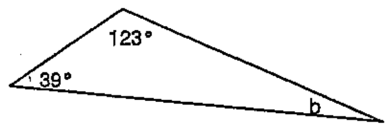

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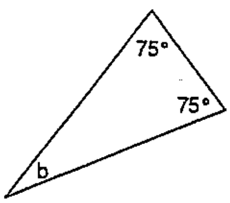

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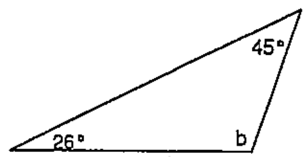
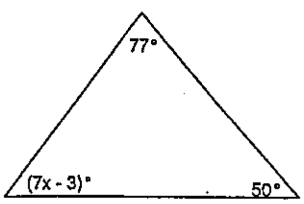
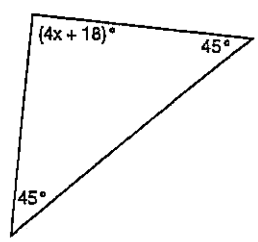


Find the value of x.

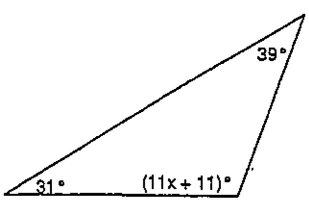
13)



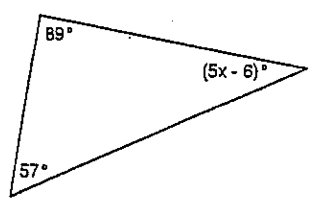
14)



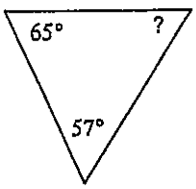
15)

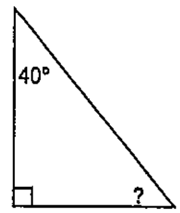


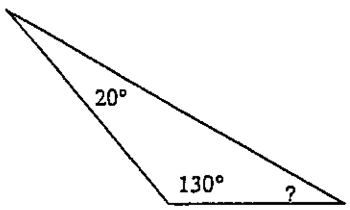
16)



Find the measure of each angle indicated.

1)


2)


3)


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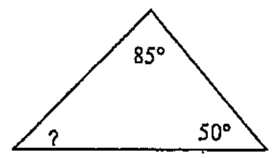
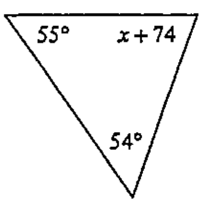
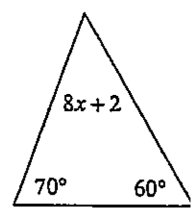
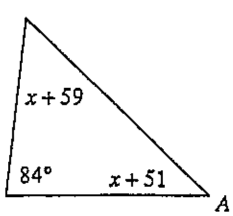


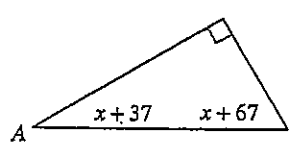
Solve for x.

17)


18)


Find the measure of angle A.

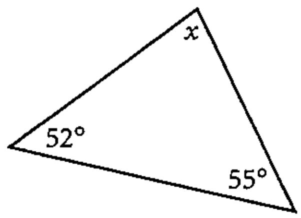
21)


22)


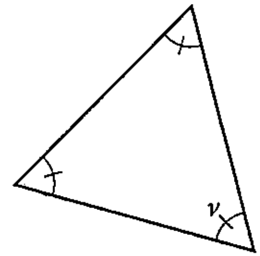
Triangle Sum Theorem

Use the Triangle Sum Conjecture to determine each lettered angle measure.

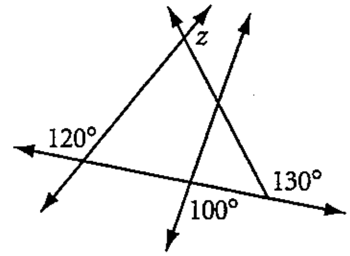
2. $x = \underline{?}$



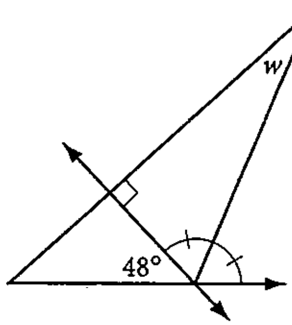
3. $v = \underline{?}$



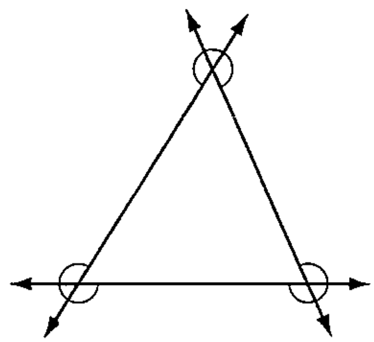
4. $z = \underline{?}$ ⓗ



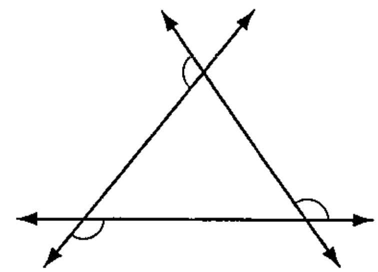
5. $w = \underline{?}$



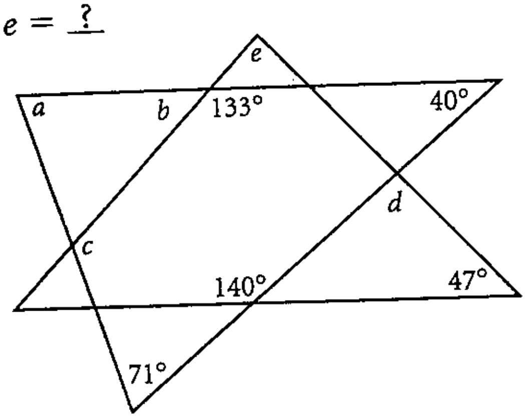
6. Find the sum of the measures of the marked angles. ⓗ



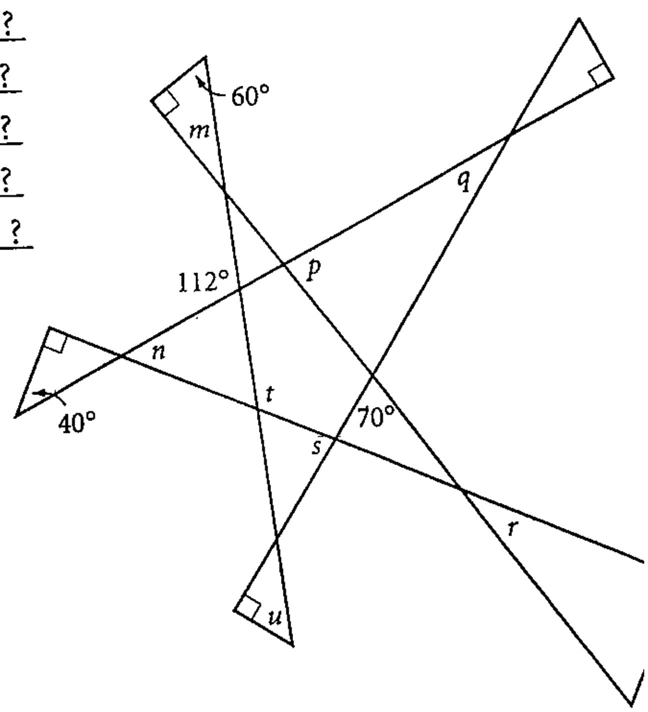
7. Find the sum of the measures of the marked angles. ⓗ



8. $a = \underline{?}$ ⓗ
$b = \underline{?}$
$c = \underline{?}$
$d = \underline{?}$
$e = \underline{?}$



9. $m = \underline{?}$
$n = \underline{?}$
$p = \underline{?}$
$q = \underline{?}$
$r = \underline{?}$
$s = \underline{?}$
$t = \underline{?}$
$u = \underline{?}$



Activity 3.1.1 Sum of Interior Angles of Quadrilaterals

Quadrilateral Angle Sum Conjecture: The sum of the interior angle measures in every quadrilateral is ____?

On a sheet of blank paper, draw any quadrilateral you like and label the vertices of the quadrilateral *A*, *B*, *C*, *D*. Measure the four angles of your quadrilateral. Then measure the four angles of your partner's quadrilateral

Your quadrilateral:

Angle measure at vertex *A*: ______

Angle measure at vertex *B*: ______

Angle measure at vertex *C*: ______

Angle measure at vertex *D*: ______

Sum of the four angle measures: ________

Your partner's quadrilateral:

Angle measure at vertex *A*: ______

Angle measure at vertex *B*: ______

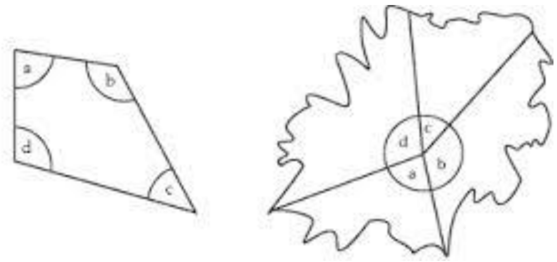
Angle measure at vertex *C*: ______

Angle measure at vertex *D*: ______

Sum of the four angle measures: ________

What conclusions can you make? Do you think this works for all quadrilaterals? Write down some of your thoughts. Discuss this with your partner and write your combined thoughts in the space provided.

Now take the quadrilateral and cut it out. Tear off the four corners. Rearrange the pieces to form a circle. Does this confirm your conjecture or conclusions from the previous exercise? Are you sure?



Review your thoughts on the quadrilateral angle sum conjecture:

Quadrilateral Angle Sum Conjecture: The sum of the interior angle measures in every quadrilateral is ____?

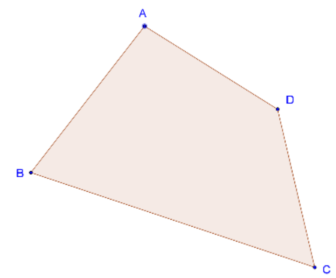
Activity 3.1.3a Proving the Quadrilateral Sum Theorem

Fill in the blanks for the proof of the Quadrilateral Sum Theorem.

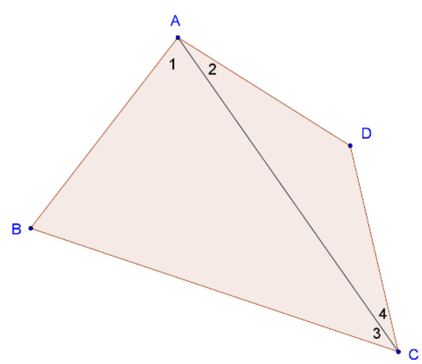
Quadrilateral Sum Theorem: The sum of the interior angle measures in any quadrilateral is 360°.

Given: $ABCD$ is a quadrilateral

Prove: $m\angle A + m\angle B + m\angle C + m\angle D = 360°$



Draw diagonal $\overline{AC}$. We can do this because ____



$m\angle B + m\angle 1 + m\angle 3 =$ _____°. We know this because __________________________.

$m\angle D + m\angle 2 + m\angle 4 =$ _____°. We know this because __________________________.

$m\angle B + m\angle 1 + m\angle 3 + m\angle D + m\angle 2 + m\angle 4 =$ _____°. We know this because

__________________________.

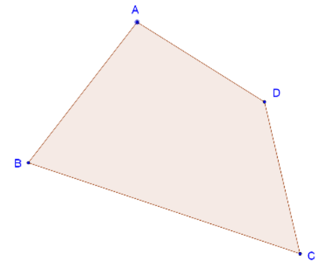
$m\angle 1 + m\angle 2 = m\angle$_____, because ____________________

$m\angle 3 + m\angle 4 = m\angle$_____, because ____________________

$m\angle B + m\angle D + m\angle BAD + m\angle BCD =$ _____°, because __________________

Activity 3.1.3b Proving the Quadrilateral Sum Theorem

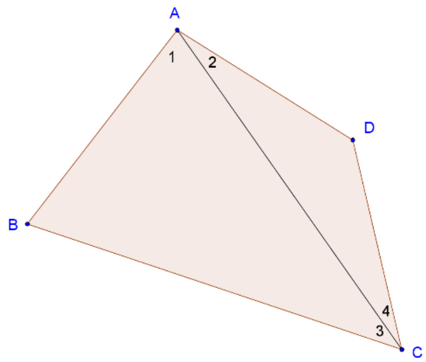
Quadrilateral Sum Theorem: The sum of the interior angle measures in any quadrilateral is 360°.



Given: *ABCD* is a quadrilateral

Prove: $m\angle A + m\angle B + m\angle C + m\angle D = 360°$

Draw diagonal $\overline{AC}$. We can do this because ______________________________.



Now complete the proof using your knowledge of the triangle angle sum theorem and the above diagram to assist you.

Lesson 1.4 • Polygons

For Exercises 1–8, complete the table.

Polygon name	Number of sides	Number of diagonals
1. Triangle		
2.		2
3.	5	
4. Hexagon		
5. Heptagon		
6.	8	
7.		35
8.	12	

For Exercises 9–11, sketch and label each figure. Mark the congruences.

9. Concave pentagon *PENTA*, with external diagonal $\overline{ET}$, and $\overline{TA} \cong \overline{PE}$.

10. Equilateral quadrilateral *QUAD*, with $\angle Q \ncong \angle U$.

11. Regular octagon *ABCDEFGH*.

For Exercises 12–15, sketch and use hexagon *ABCDEF*.

12. Name the diagonals from *A*.

13. Name a pair of consecutive sides.

14. Name a pair of consecutive angles.

15. Name a pair of non-intersecting diagonals.

For Exercises 16–19, use these figures at right.

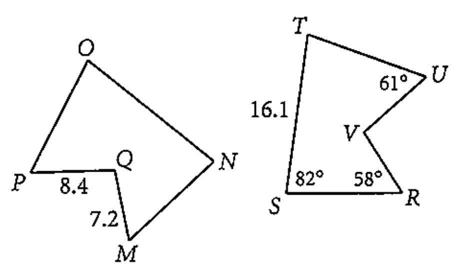
$MNOPQ \cong RSTUV$

16. $m\angle N =$ _____

17. $VR =$ _____

18. $m\angle P =$ _____

19. $ON =$ _____



20. How many different (noncongruent) convex quadrilaterals can you make on a 3-by-3 dot grid, using the dots as vertices?

Activity 3.1.4 The Polygon Angle Sum Theorem

1. In Activity 3.1.2, you proved the Triangle Angle Sum Theorem. State the theorem here in your own words.

____

____.

2. In Activity 3.1.3, you proved the Quadrilateral Angle Sum Theorem. State the theorem here in your own words.

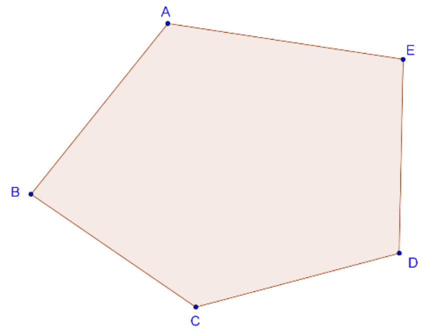
____

____.

Now you are going to work on the Polygon Angle Sum Theorem. Recall that a **diagonal** is a line segment connecting nonconsecutive vertices in a polygon. You proved the Quadrilateral Sum Theorem by drawing a diagonal.

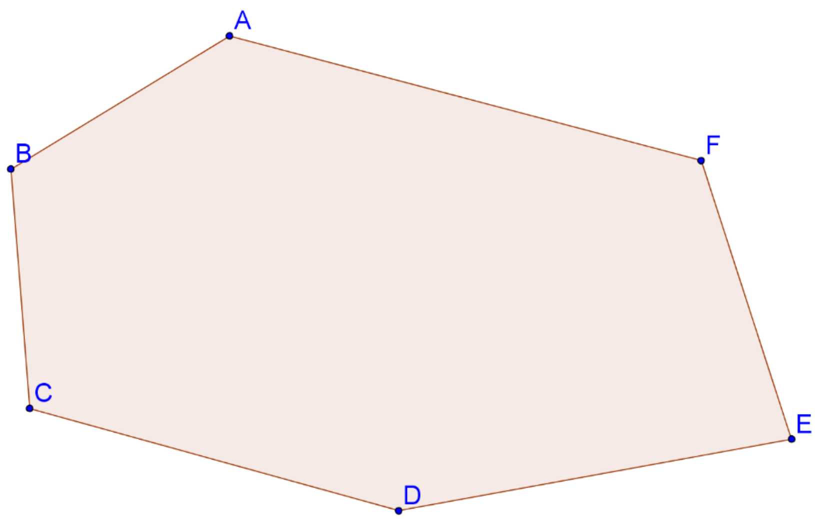
Our work will be demonstrated primarily with convex polygons. A polygon is **convex** if each of its diagonals lies completely within the polygon. After completing this activity you may want to discuss whether this theorem also applies to non-convex polygons.

3. On the figure below, draw diagonal $\overline{AC}$. What type of polygons did you create within polygon *ABCDE*? __________________________ and____________________________. What is the sum of the angle measures for each of the polygons you listed? ______and_____. Using this information, what is the sum of the interior angles of this pentagon? __________.



4. Do you think this would be true for all convex pentagons? Explain.

5. On the figure below, draw diagonal $\overline{AC}$. What type of polygons did you create within polygon *ABCDEF*? ___________________________ and __________________________. What is the sum of the angle measures for each of the polygons you listed? _________and __________. Using this information, what is the sum of the interior angles of this hexagon? __________.



6. Do you think this would be true for all hexagons? Explain.

7. Use the table below to help you make a conjecture about the Polygon Angle Sum Theorem.

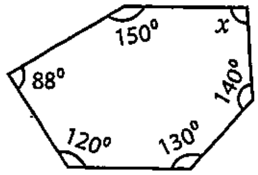
Number of Sides in the Convex Polygon	Name of the Polygon	Sum of the Interior Angle Measures of the Polygon
3	Triangle	180°
4	Quadrilateral	360°
5		
6	Hexagon	
7		900°
8		
9	Nonagon	
10		1440°
n	*n*-gon	

8. What do you notice about the differences between pairs of values in the right hand column?

9. Hypothesize a formula for the sum of the interior angles of a polygon. Use the graph below to display the data. Label the horizontal axis as “Number of sides”. Label the vertical axis as “Sum of interior angles”. Label axes appropriately and graph the data.

Interior Angle 1

Example:

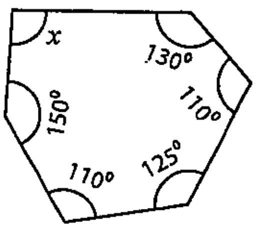


Sum of the interior angles = (Number of sides - 2) x 180°
= (6 - 2) x 180°
= 4 x 180 = **720°**

Sum of the interior angles = 120° + 140° + 130° + 150° + 88° + x
720° = 628° + x
x = **720°** - 628° = **92°**

Find the interior angle for each irregular polygon.

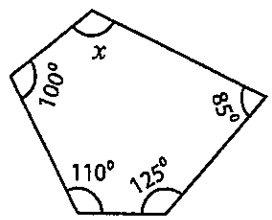
1)



Sum of the interior angles =

x =

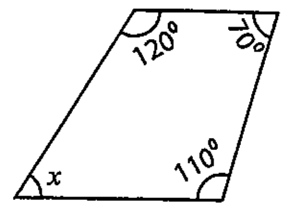
2)



Sum of the interior angles =

x =

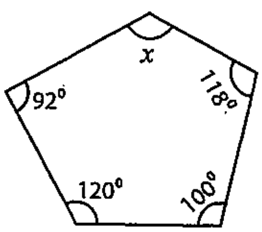
3)



Sum of the interior angles =

x =

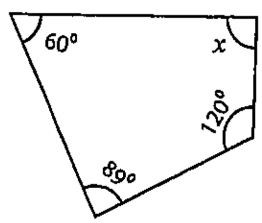
4)



Sum of the interior angles =

x =

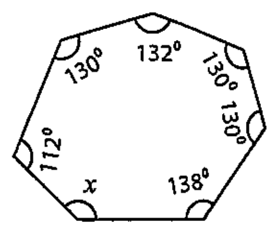
5)



Sum of the interior angles =

x =

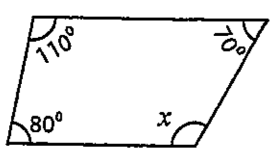
6)



Sum of the interior angles =

x =

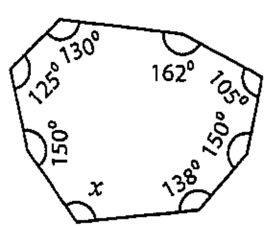
7)



Sum of the interior angles =

x =

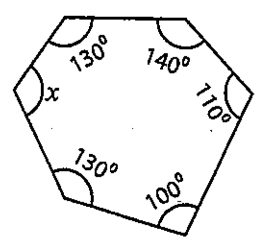
8)



Sum of the interior angles =

x =

9)

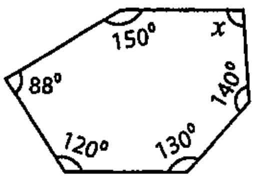


Sum of the interior angles =

x =

Interior Angle

Example:



Sum of the interior angles = (Number of sides - 2) x 180°

= (6 - 2) x 180°

= 4 x 180 = **720°**

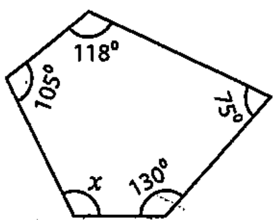
Sum of the interior angles = 120° + 140° + 130° + 150° + 88° + x

720° = 628° + x

x = **720°** - 628° = **92°**

Find the interior angle for each irregular polygon.

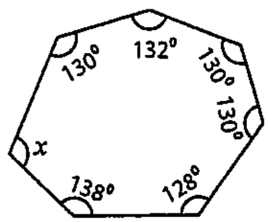
1)



Sum of the interior angles = ______

x = ______

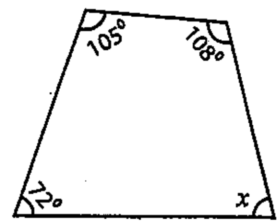
2)



Sum of the interior angles = ______

x = ______

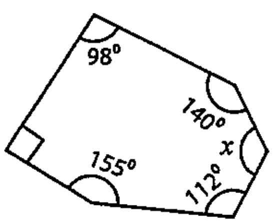
3)



Sum of the interior angles = ______

x = ______

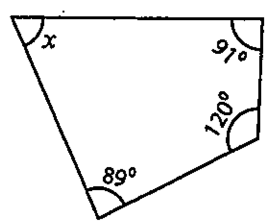
4)



Sum of the interior angles = ______

x = ______

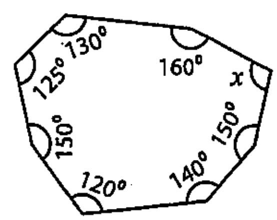
5)



Sum of the interior angles = ______

x = ______

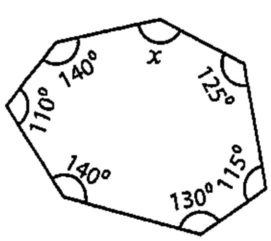
6)



Sum of the interior angles = ______

x = ______

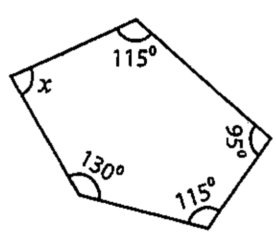
7)



Sum of the interior angles = ______

x = ______

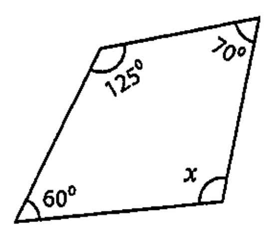
8)



Sum of the interior angles = ______

x = ______

9)



Sum of the interior angles = ______

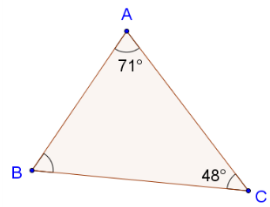
x = ______

Activity 3.1.5 Applications of the Triangle Angle Sum Theorems

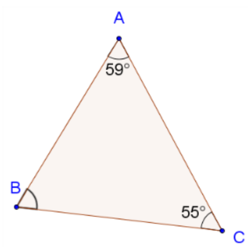
Part I – Triangles

Determine the missing degree measure for each diagram. m $\angle B$

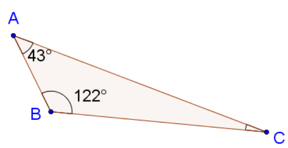
1. m $\angle B$ = ________



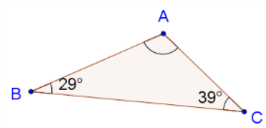
2. m $\angle B$ = ________



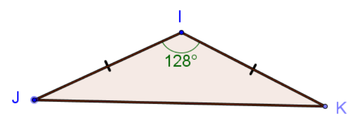
3. m $\angle C$ = ________



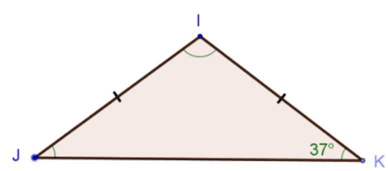
4. m $\angle A$ = ________



5. m $\angle J$ = ________



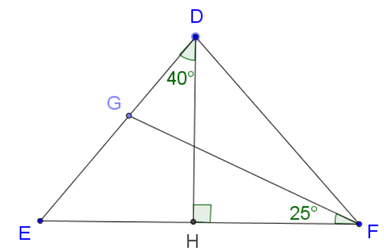
6. m $\angle I$ = ________



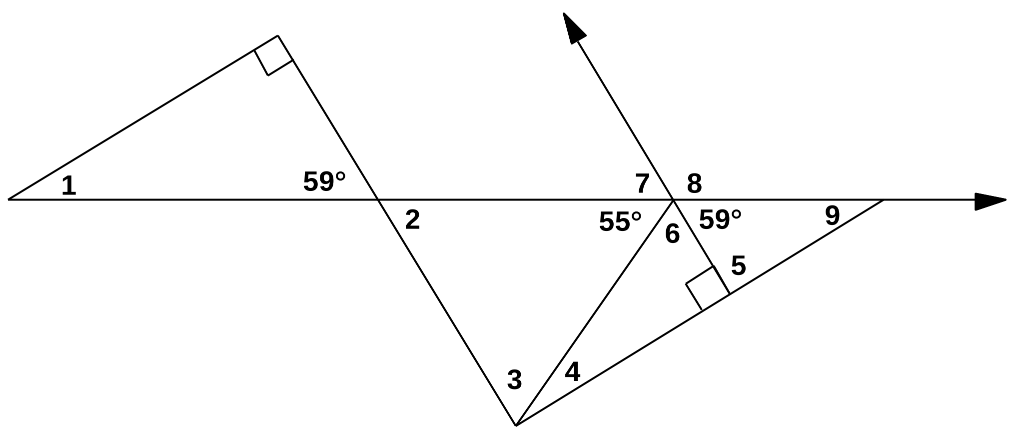
Use the diagram at the right for questions 7 and 8.

7. m $\angle GEH$ = ________

8. m $\angle DGF$ = ________



9. Use the diagram below to determine the measures of the angles.



m ∠1 = ________

m ∠2 = ________

m ∠3 = ________

m ∠4 = ________

m ∠5 = ________

m ∠6 = ________

m ∠7 = ________

m ∠8 = ________

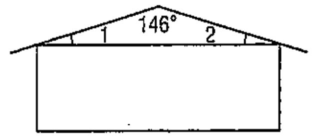
m ∠9 = ________

4-2 Skills Practice

Angles of Triangles

Find the measure of each numbered angle.

1. 

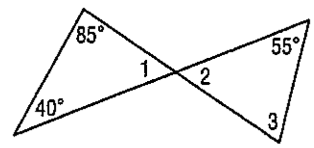
2. 

Find each measure.

3. $m\angle 1$

4. $m\angle 2$

5. $m\angle 3$

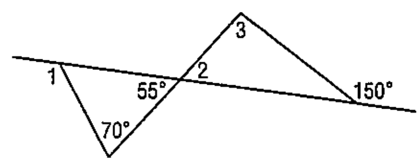


Find each measure.

6. $m\angle 1$

7. $m\angle 2$

8. $m\angle 3$



Find each measure.

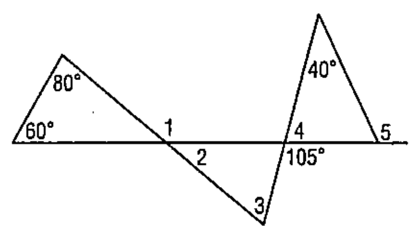
9. $m\angle 1$

10. $m\angle 2$

11. $m\angle 3$

12. $m\angle 4$

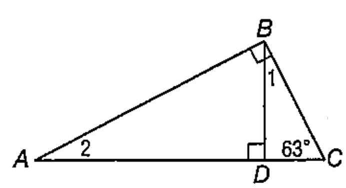
13. $m\angle 5$



Find each measure.

14. $m\angle 1$

15. $m\angle 2$

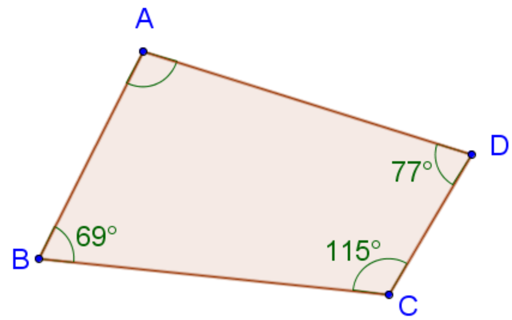


Activity 3.1.5 Applications of the Quadrilateral and Polygon Angle Sum Theorems

Part II -- Quadrilaterals and Other Polygons

1. Determine the measure of $\angle A$.

m $\angle A$ = ________



2. Determine the value of x. Then determine the measures of all of the angles in the figure.

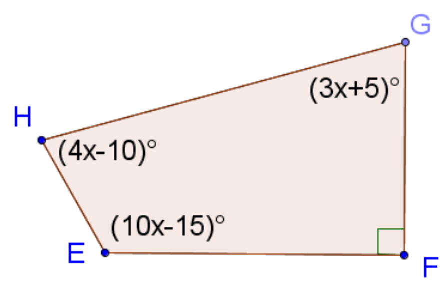
x = ________

m $\angle E$ = ________

m $\angle F$ = ________

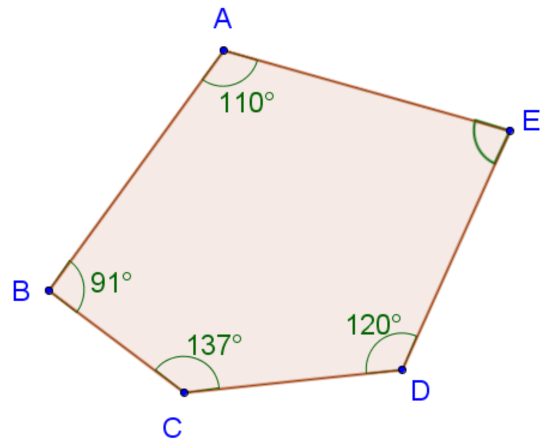
m $\angle G$ = ________

m $\angle H$ = ________



3. Determine the measure of $\angle E$.

m $\angle E$ = ________



4. Determine the value of *x*. Then determine the measures of all of the angles in the figure.

x = ________

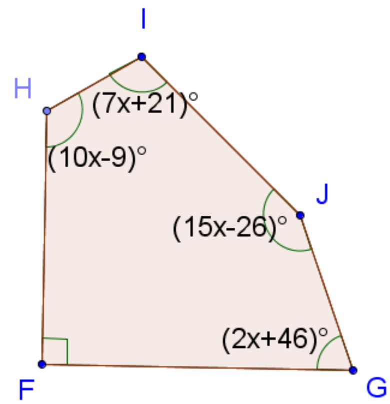
m $\angle F$ = ________

m $\angle G$ = ________

m $\angle H$ = ________

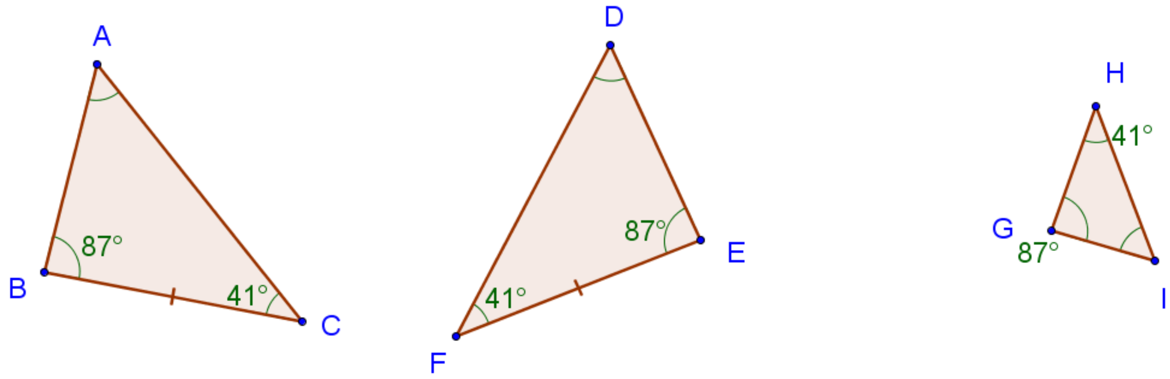
m $\angle I$ = ________

m $\angle J$ = ________



Activity 3.1.6 Angle-Angle-Side Congruency

1. Determine the measure of A. m$\angle A$ = ______.
2. Determine the measure of D. m$\angle D$ = ______.
3. Determine the measure of I. m$\angle I$ = ______.



4. Can you make any conclusions? Write your thoughts down about the relationships between the triangles and the given information.

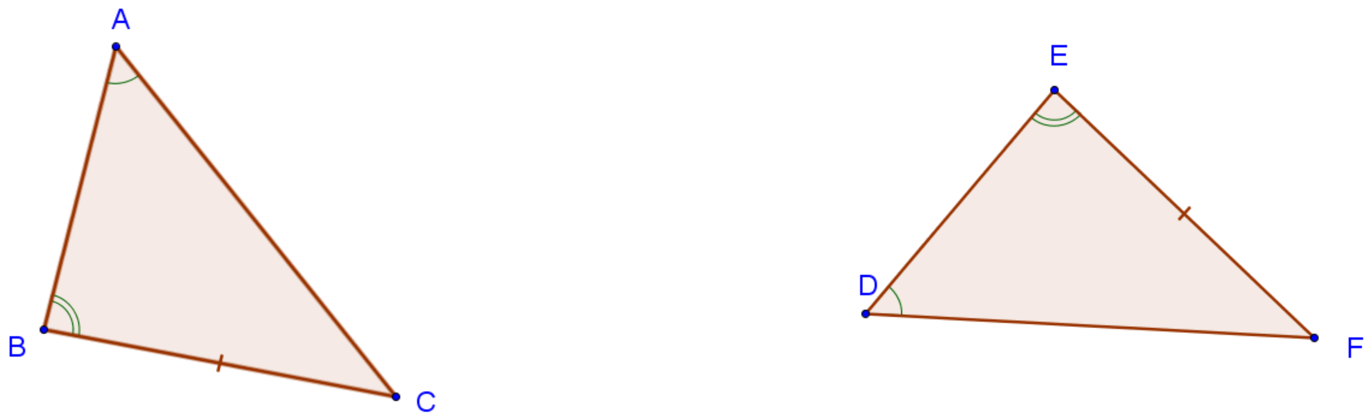
5. Discuss your ideas with a classmate and try to formalize your ideas into a theorem that can be communicated with the rest of the class.

6. If _____ angles from one triangle are equal to _____ angles from another triangle, then their third angles are ______.

7. Using this new theorem formed in question 6, how can we prove congruence between the following triangles?

Prove: $\Delta ABC \cong \Delta DEF$

Given: $\angle A = \angle D, \angle B = \angle E,\ BC = EF$



{HINT: Use the theorem on the previous page to help you obtain the information necessary for a previously proven method of congruence}

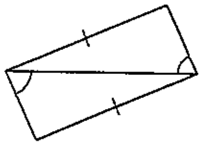
You have now proved the AAS Congruence Theorem: It two angles and a non-included _____ of one triangle are congruent to two ______ and the corresponding non-included ______of a second triangle, then the two triangles are congruent.

3.1.6A

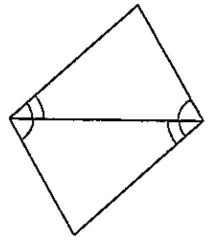
SSS, SAS, ASA, and AAS Congruence

State if the two triangles are congruent. If they are, state how you know.

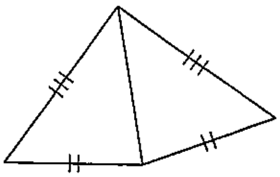
1)



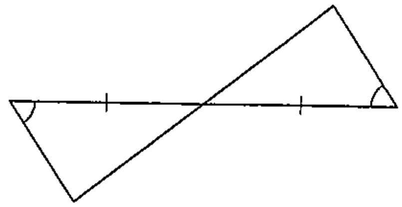
2)



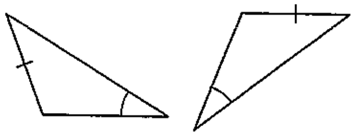
3)



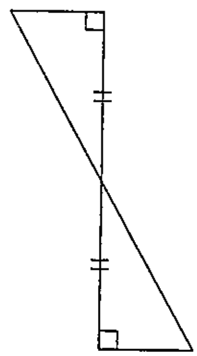
4)



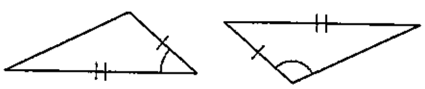
5)



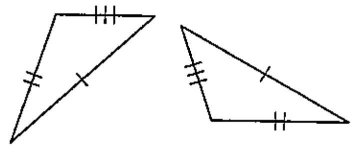
6)



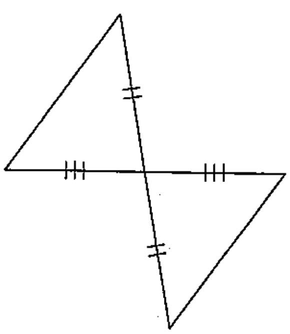
7)



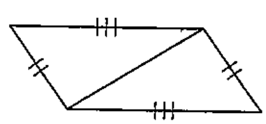
8)



9)

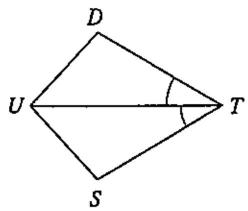


10)

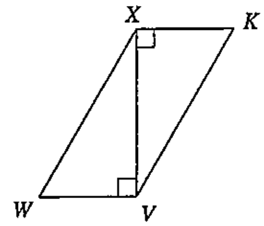


State what additional information is required in order to know that the triangles are congruent for the reason given.

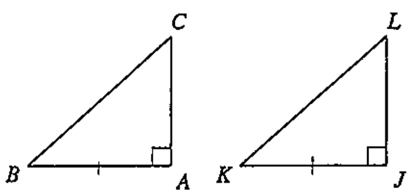
11) ASA



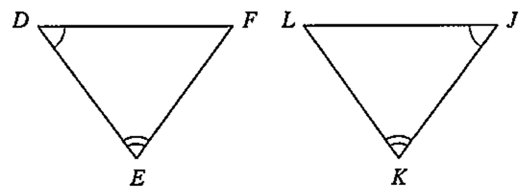
12) SAS



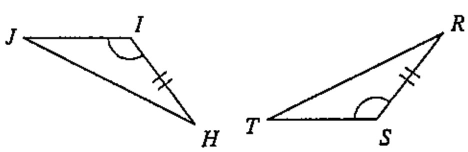
13) SAS



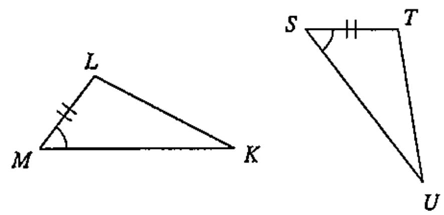
14) ASA



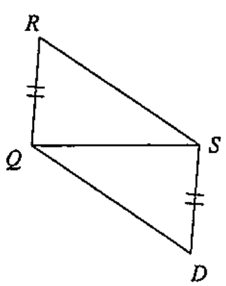
15) SAS



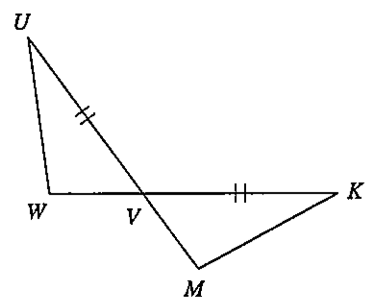
16) ASA



17) SSS



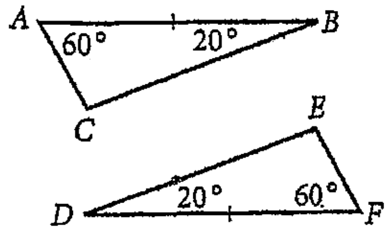
18) SAS



Triangle Congruence

Decide whether each pair of triangles can be proven congruent. If so, write an appropriate congruence statement and tell which postulate you used. If not, write *Cannot Be Determined.*

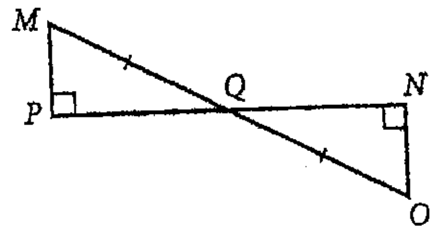
1)



ΔABC ≅ ____________

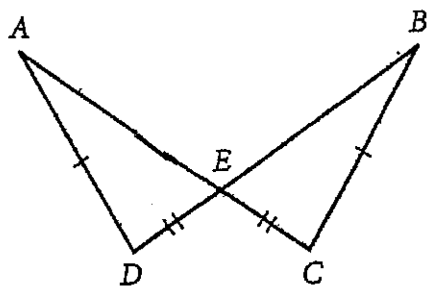
____

2)



ΔMPQ ≅ ____________

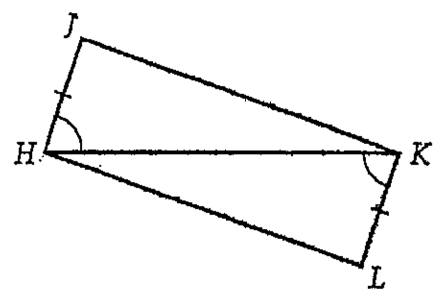
____



ΔAED ≅ ____________

____

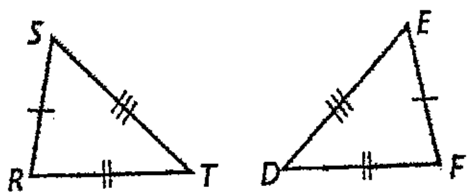
4)



ΔJHK ≅ ____________

____

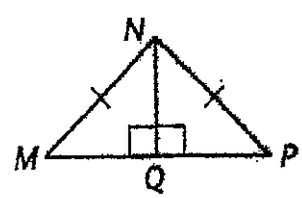
5)



ΔSRT ≅ ____________

____

6)



ΔMNQ ≅ ____________

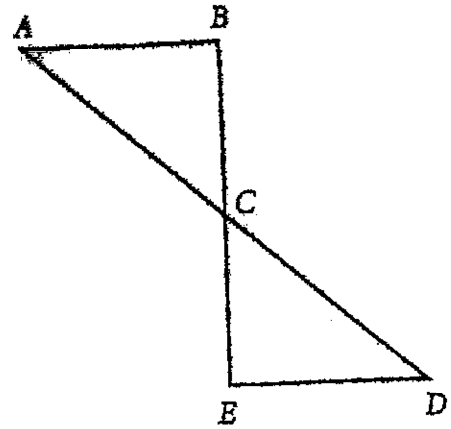
____

7) $\overline{BE}$ bisects $\overline{AD}$

$\angle B$ and $\angle E$ are right angles

$\triangle ABC \cong$ ____________________

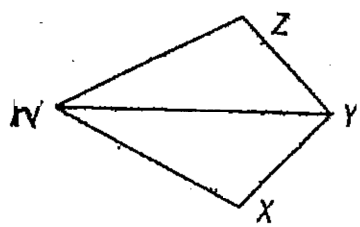
____



8) $\overline{ZY} \cong \overline{YX}$; $\overrightarrow{YW}$ bisects $\angle ZYX$

$\triangle WZY \cong$ ____________________

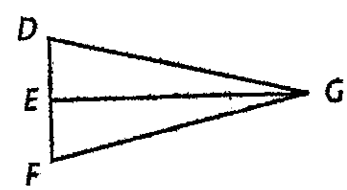
____



9) $\overline{GE} \perp \overline{DF}$; E is the midpoint of $\overline{DF}$

$\triangle DEG \cong$ ____________________

____

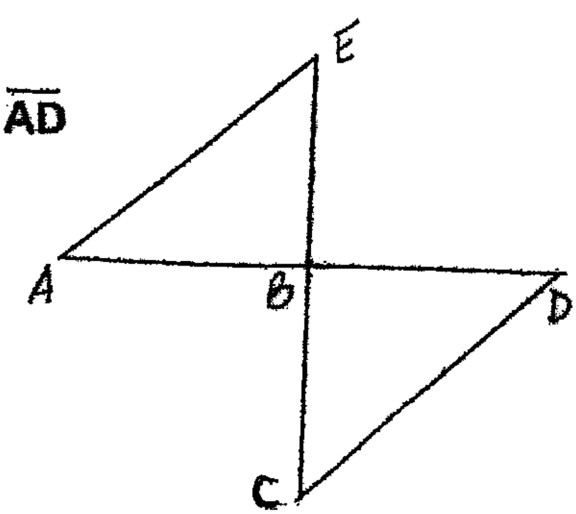


10) $\overline{EC} \perp \overline{AD}$; $\overline{AE} \cong \overline{DC}$;

B is the midpoint of $\overline{EC}$ and $\overline{AD}$

$\triangle AEB \cong$ ____________________

____



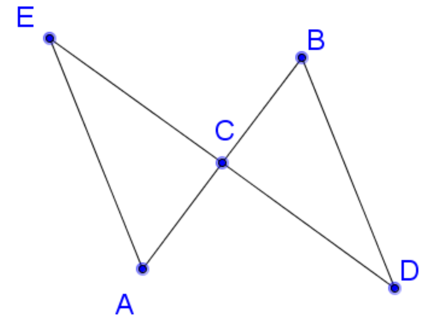
Activity 3.1.6 Angle-Angle-Side Congruency

Proving Congruency using SAS, ASA, SSS, and AAS

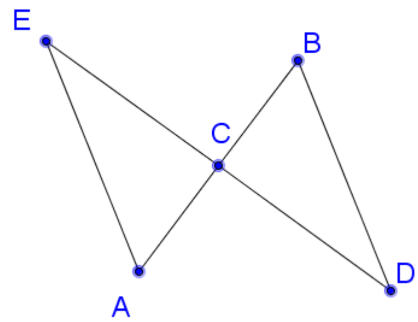
1. Can the following congruence statement be proved using the given information and the diagram? If so, prove the congruence statement. If not, state why not.

Prove: $\Delta CBD \cong \Delta CAE$

Given: $\angle A = \angle B$ and C is the midpoint of segment $\overline{AB}$.



2. Can the following congruence statement be proved using the given information and the diagram? If so, prove the congruence statement. If not, state why not.



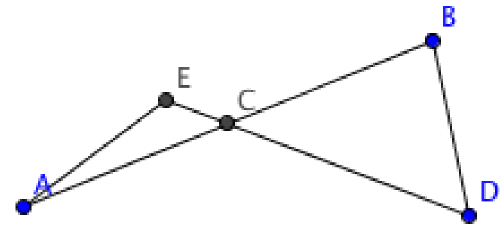
Prove: $\Delta CBD \cong \Delta CAE$

Given: $\angle A = \angle B$ and $BD = AE$

3. Can the following congruence statement be proved using the given information and the diagram? If so, prove the congruence statement. If not, state why not

Prove: $\Delta CBD \cong \Delta CAE$

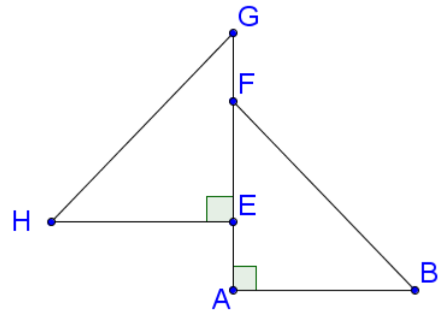
Given: C is the midpoint of segment $\overline{AB}$ and $BD = AE$



4. Can the following congruence statement be proved using the given information and the diagram? If so, prove the congruence statement. If not, state why not

Prove: $\Delta ABF \cong \Delta EHG$

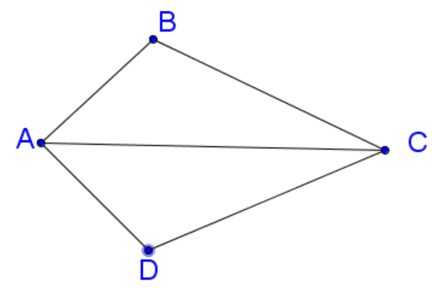
Given: m $\angle H =$ m $\angle B$ and $FG = AE$; angles *HEG* and *FAB* are right angles.



5. Can the following congruence statement be proved using the given information and the diagram? If so, prove the congruence statement. If not, state why not.

Prove: $\Delta ABC \cong \Delta ADC$

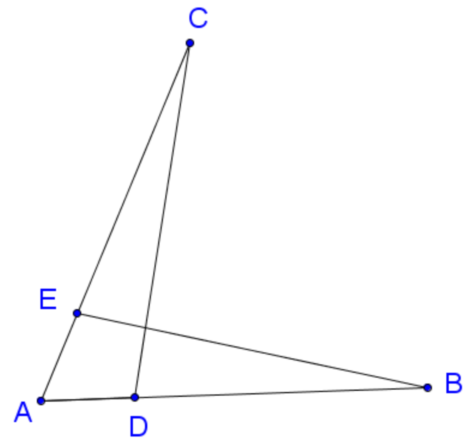
Given: Segment $\overline{AC}$ bisects $\angle BAD$ and
Segment $\overline{AC}$ bisects $\angle BCD$



6. Can the following congruence statement be proved using the given information and the diagram? If so, prove the congruence statement. If not, state why not.

Prove: $\angle B \cong \angle C$

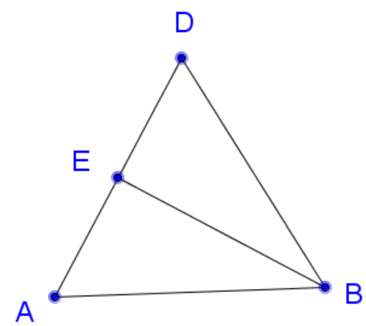
Given: $AC = AB$ and $EA = DA$



7. Can the following congruence statement be proved using the given information and the diagram? If so, prove the congruence statement. If not, state why not.

Given: ΔABD is an equilateral triangle and E is the midpoint of segment $\overline{AD}$

Prove: $\overline{EB}$ bisects $\angle DBA$.



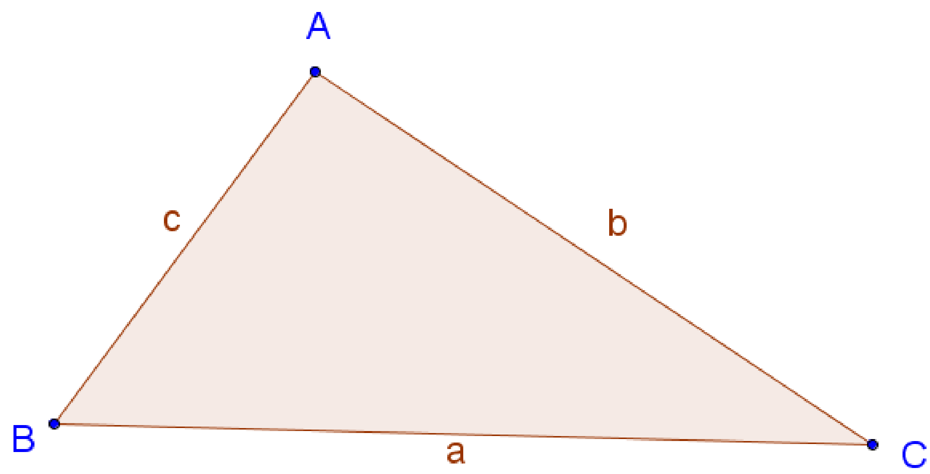
8. In question 14 above prove that $\overline{EB} \perp \overline{AD}$.

Activity 3.2.1a Discovering Inequalities in Triangles

Use your protractor and ruler to measure the angles and side lengths of the following triangles. Fill in the table above each triangle. Once you are finished measuring all four triangles, review your data. What relationships do you notice about the lengths of the sides in comparison to the angles in the triangles? Is there any relationship you notice regarding the three side lengths?

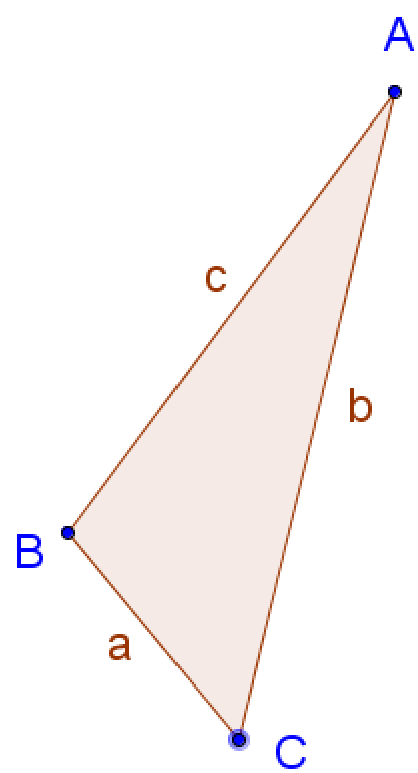
1.

SIDES (in mm)		ANGLES	
a		A	
b		B	
c		C	



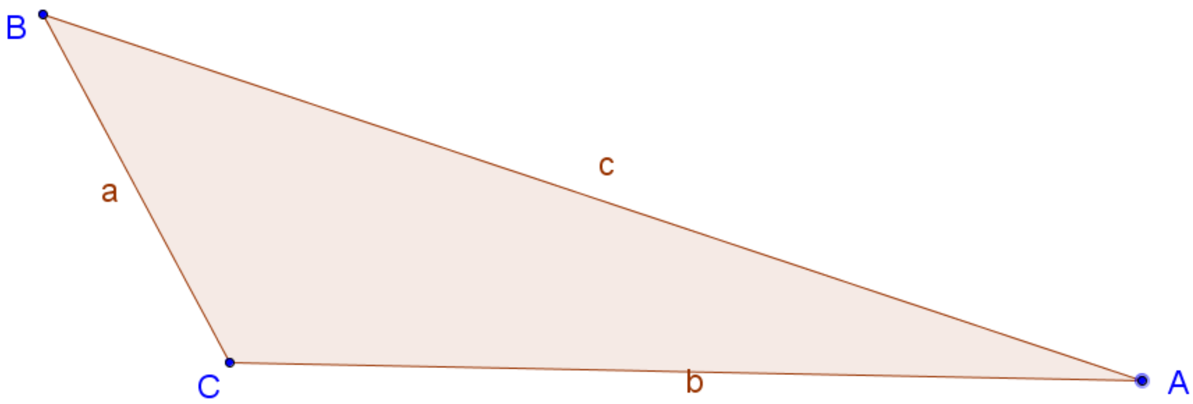
2.

SIDES (in mm)		ANGLES	
a		A	
b		B	
c		C	



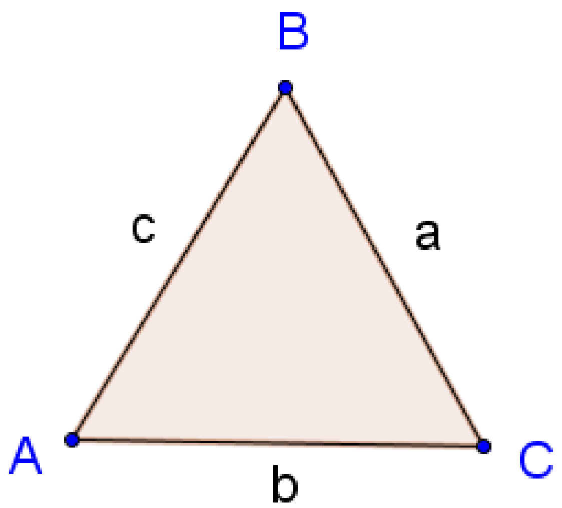
3.

SIDES (in mm)		ANGLES	
a		A	
b		B	
c		C	



4.

SIDES (in mm)		ANGLES	
a		A	
b		B	
c		C	



CONJECTURE TIME

5. a. In each triangle, find the longest side and the largest angle. What do you notice?

   b. Now find the shortest side and the smallest angle in each triangle. What do you notice?

6. Ask a friend: Turn to your neighbor and write down their answers to question 5.

7. Are you and your neighbor in agreement? If not, try to resolve any differences.

8. In each triangle, find the sum of the lengths of the two shorter sides. Compare this sum to the length of the larger side. What do you notice?

9. Ask a friend: Turn to your neighbor and write down their answer to question 8.

10. Are you and your neighbor in agreement? If not, try to resolve any differences.

11. Combine your thoughts from your answers to questions 5 and 6 to make a final conjecture.

12. Combine your thoughts from your answers to questions 8 and 9 to make a final conjecture.

Room for Notes from class discussion on conjectures (answers to questions 11 and 12).

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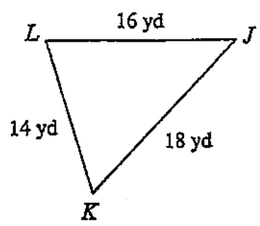
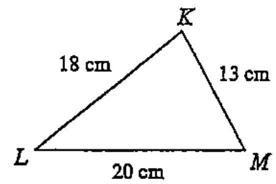
Inequalities in One Triangle

Order the angles in each triangle from smallest to largest.

1)



2)

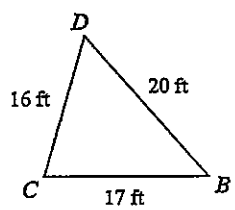


3) In ΔRQP
$QP = 15$ ft
$RP = 25$ ft
$RQ = 13$ ft

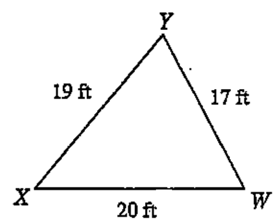
4) In ΔTUV
$UV = 17$ yd
$TV = 14$ yd
$TU = 9$ yd

Name the largest and smallest angle in each triangle.

5)



6)

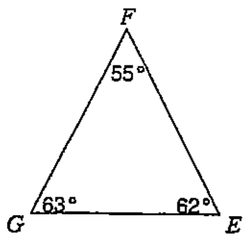


7) In ΔUVW
$VW = 13$ m
$UW = 11.7$ m
$UV = 5.8$ m

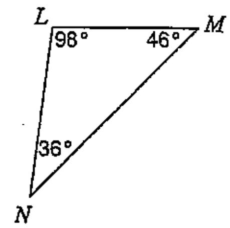
8) In ΔEFG
$FG = 10.9$ in
$EG = 17$ in
$EF = 10.9$ in

Order the sides of each triangle from shortest to longest.

9)



10)

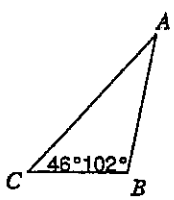


11) In ΔVWX
$m\angle V = 50°$
$m\angle W = 48°$
$m\angle X = 82°$

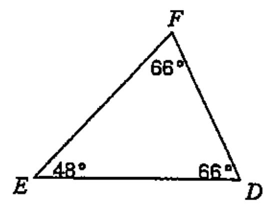
12) In ΔSTU
$m\angle S = 50°$
$m\angle T = 70°$
$m\angle U = 60°$

Name the longest and shortest side in each triangle.

13)



14)



15) In ΔDEF
$m\angle D = 35°$
$m\angle F = 95°$

16) In ΔKLM
$m\angle K = 50°$
$m\angle L = 100°$
$m\angle M = 30°$

Critical thinking questions:

17) In triangle ABC:
AB is the longest side.
70° is the measure of angle B.

Find the range of possible measures for angle A.

18) In triangle XYZ:
XY is the shortest side.
30° is the measure of angle Y.

Find the range of possible measures for angle X.

Lesson 4.3 • Triangle Inequalities 3.2.1B

In Exercises 1–4, determine whether it is possible to draw a triangle with sides of the given measures. If it is possible, write yes. If it is not possible, write no and make a sketch demonstrating why it is not possible.

1. 16 cm, 30 cm, 45 cm

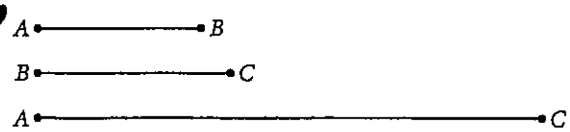
2. 9 km, 17 km, 28 km

3. 32 in., 60 in., 87 in.

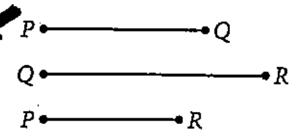
4. 13.4 ft, 17.7 ft, 31.1 ft

In Exercises 5 and 6, use a compass and straightedge to construct a triangle with the given sides. If it is not possible, explain why not.

5.



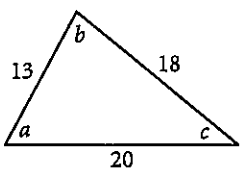
6.



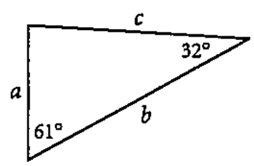
7. If 17 and 36 are the lengths of two sides of a triangle, what is the range of possible values for the length of the third side?

In Exercises 8–13, arrange the unknown measures in order from greatest to least.

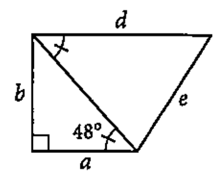
8.



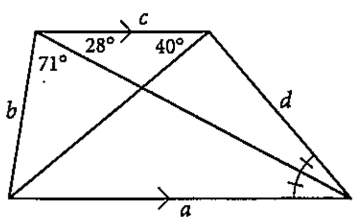
9.



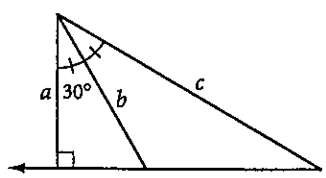
10.



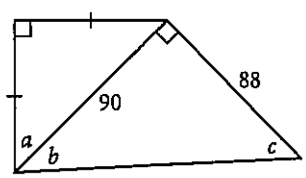
11.



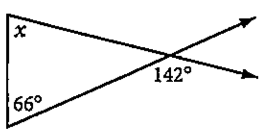
12.



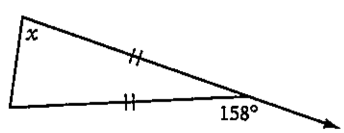
13.



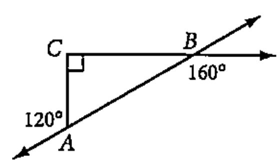
14. $x =$ ______



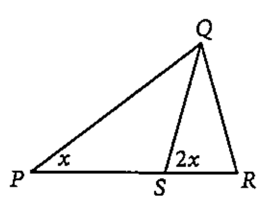
15. $x =$ ______



16. What's wrong with this picture?



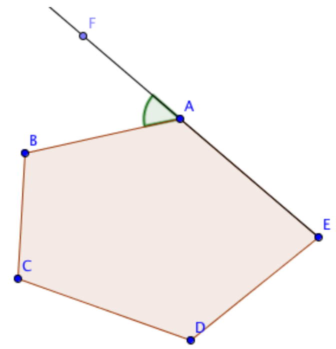
17. Explain why $\triangle PQS$ is isosceles.



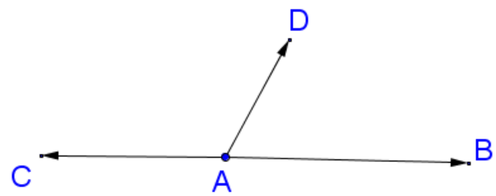
18. Explain why the sum of the three altitudes of a triangle is always less than its perimeter.

Activity 3.2.2 Exterior Angles of Polygons

When one side of a convex polygon is extended beyond a vertex, the angle formed with the adjacent side is called an **exterior angle** of the polygon. In the figure at the right $\angle FAB$ is an exterior angle of pentagon *ABCDE*.

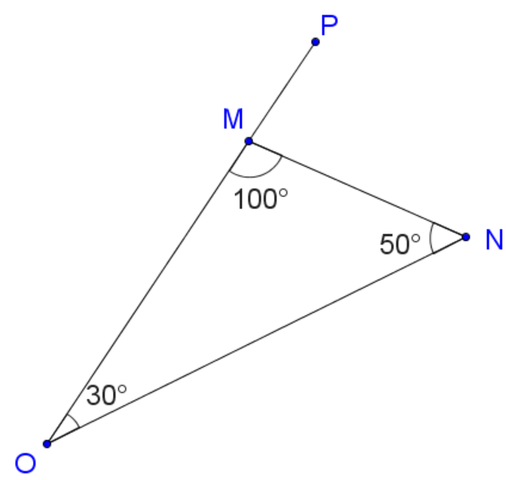


The exterior angle of a polygon and its adjacent angle form a **linear** pair. Recall that the definition of a linear pair: two angles that have a common side and whose other sides are opposite rays. In the diagram below $\angle CAD \text{ and } \angle BAD$ form a linear pair. The **Linear Pair Postulate** states that the angles in a linear pair are **supplementary**, that is, the sum of their measures is 180°.



We will use this information to make a conjecture about relationships between exterior angles and the non-adjacent angles in a triangle.

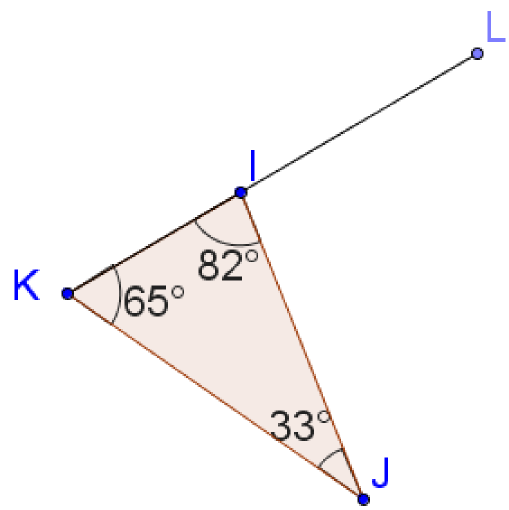
1. Using the definition of supplementary angles and linear pairs, what is the measure of $\angle NMP$?



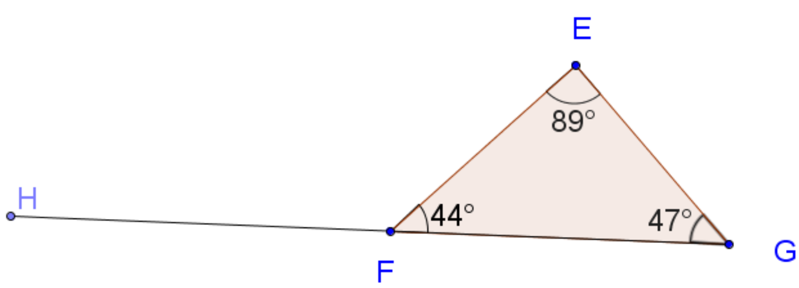
2. What do you notice about the relationship between $\angle NMP$ and angles *MON* and *ONM*?

3. Using the definition of supplementary angles and linear pairs, what is the measure of $\angle LIJ$?

4. What do you notice about the relationship between $\angle LIJ$ and angles *IKJ* and *IJK*?



5. Using the definition of supplementary angles and linear pairs, what is the measure of $\angle EFH$?



6. What do you notice about the relationship between $\angle EFH$ and angles *FEG* and *EGF*?

7. Based on what you have observed, make one or more conjectures.

8. Ask a friend: Turn to your neighbor and write down their answer to question 7?

9. Are you and your neighbor in agreement? If not, try to resolve any differences.

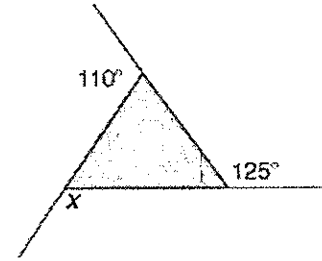
10. Combine your thoughts from your answers to questions 7 and 8 to make final conjectures.

11. After discussion with the entire class, prove your conjecture(s).

Example Determine the angle measure indicated by x.

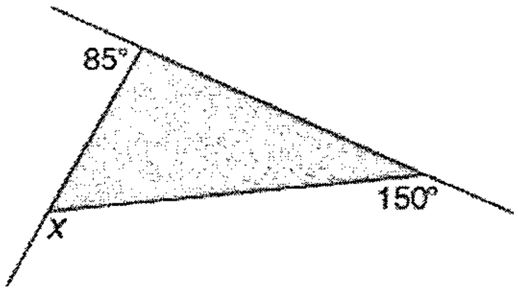
Solution The sum of the 3 exterior angles of a triangle is 360°.

$$\text{So, } x + 110° + 125° = 360°$$
$$x + 235° = 360°$$
$$x = 360° - 235°$$
$$= 125°$$

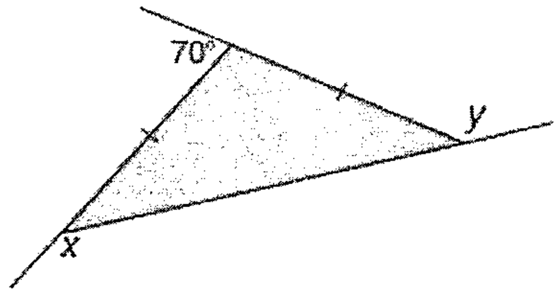


**Part I - Calculate the missing <u>interior</u> and <u>exterior</u> angles.

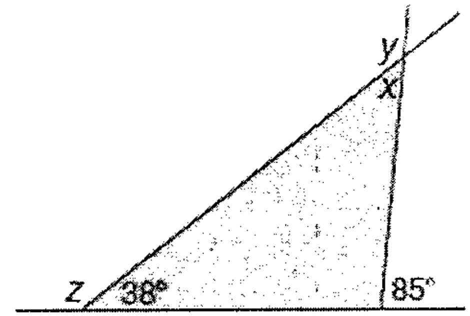
a)



b)



c)



a) The 3 **interior** angles are ______°, ______°, & ______°.

The 3 **exterior** angles are ______°, ______°, & ______°.

b) The 3 **interior** angles are ______°, ______°, & ______°.

The 3 **exterior** angles are ______°, ______°, & ______°.

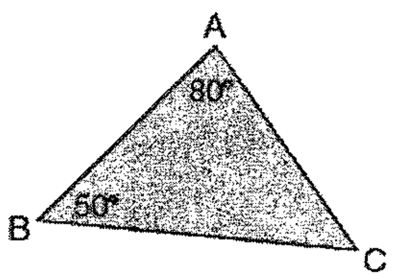
c) The 3 **interior** angles are ______°, ______°, & ______°.

The 3 **exterior** angles are ______°, ______°, & ______°.

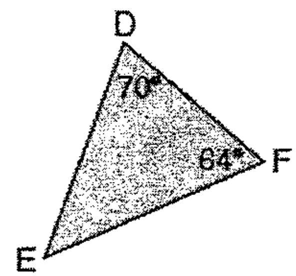
Part II – **<u>Draw</u> the 3 exterior angles by extending the sides of the triangle as was done in (a) on the other side.

Calculate the missing interior and exterior angles.

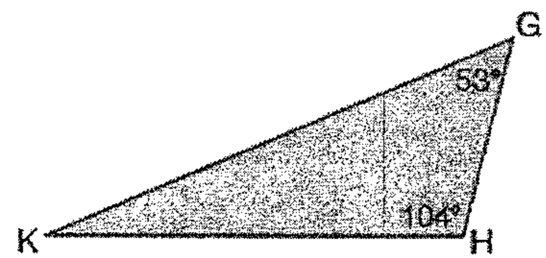
a)



b)



c)



a) The 3 **interior** angles are _____°, _____°, & _____°. The 3 **exterior** angles are _____°, _____°, & _____°.	b) The 3 **interior** angles are _____°, _____°, & _____°. The 3 **exterior** angles are _____°, _____°, & _____°.	c) The 3 **interior** angles are _____°, _____°, & _____°. The 3 **exterior** angles are _____°, _____°, & _____°.

Activity 3.2.5 Possible or Impossible?

Determine whether each triangle is possible or impossible to occur. Circle the appropriate word. Then, state your reasons using vocabulary and/or theorems from the unit to support your answer.

1. Possible / Impossible

Reason: ______________________________

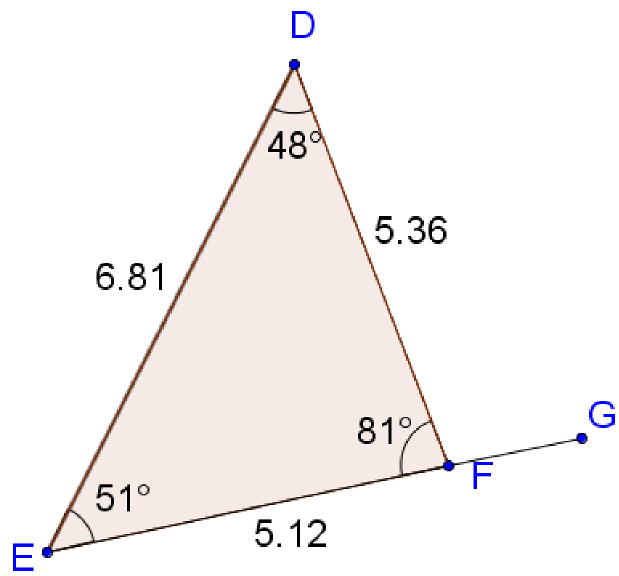
____

____

____

____

____



2. Possible / Impossible

Reason: ______________________________

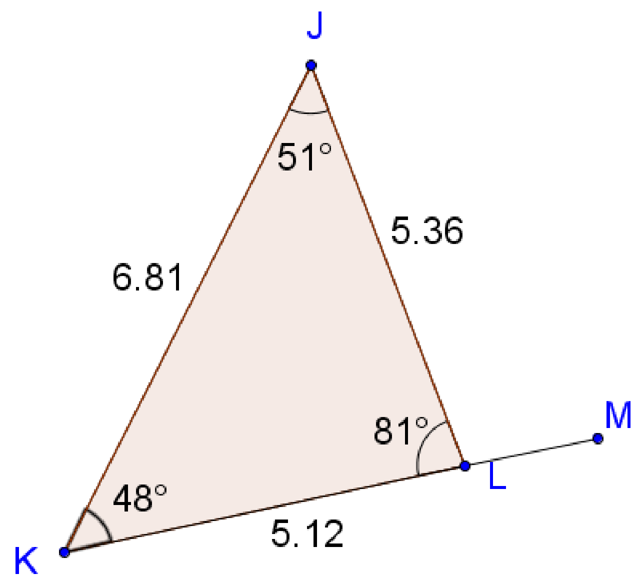
____

____

____

____

____



3. Possible / Impossible

Reason: ______________________________

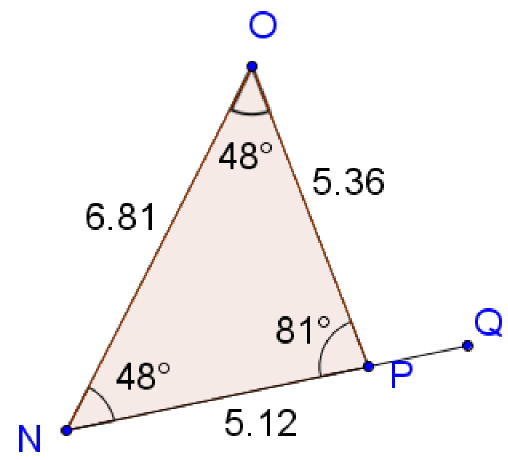
____

____

____

____

____



4. Possible / Impossible

Reason: ______________________________

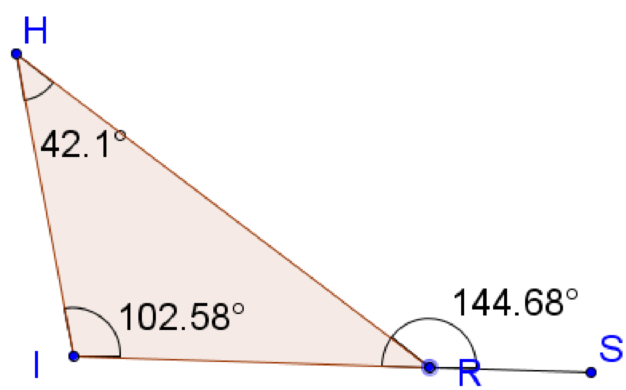
____

____

____

____

____



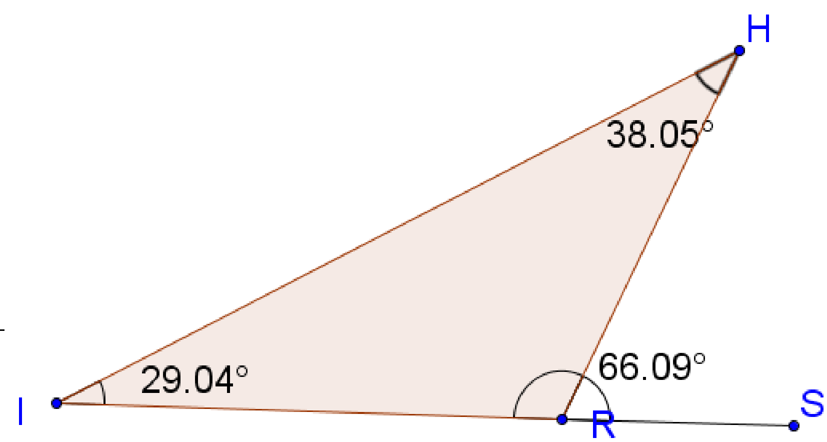
5. Possible / Impossible

Reason: ______________________

____

____

____



____

____

6. Possible / Impossible

Reason: ______________________

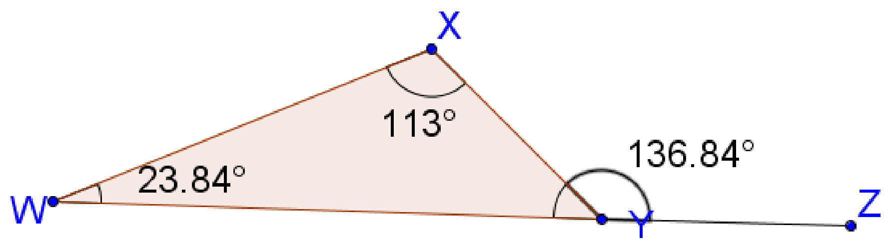
____

____

____

____

____

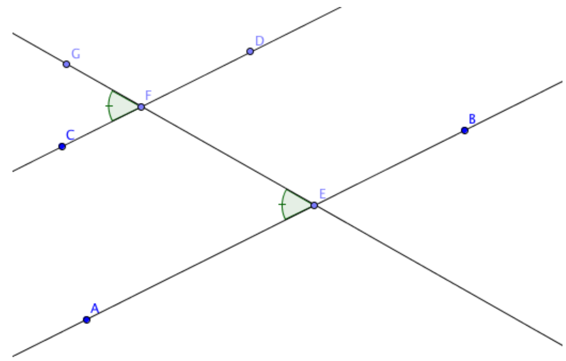


Activity 3.3.2 Parallel Lines Corresponding Angles Converse

1. Prove the **Parallel Lines Corresponding Angles Converse**: If two lines are cut by a transversal and a pair of corresponding angles are congruent, then the lines are parallel. Fill in the blanks.

Given: Transversal $\overleftrightarrow{EF}$ intersects lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ at points *E* and *F*.

m ∠ *CFG* = m ∠ *AEF*.

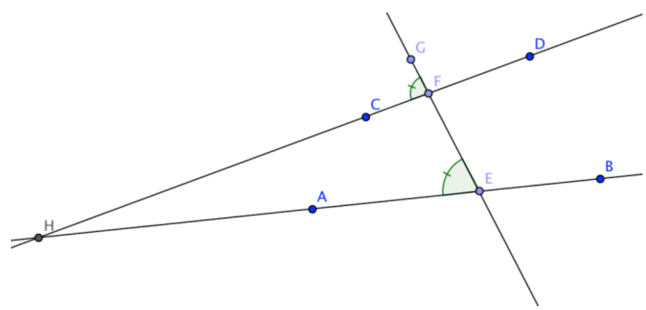


Prove: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Proof:

$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ either intersect or they are parallel.

Why? ___________________________



Suppose $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at a point we will call "*H*."

Then *E*, *F*, and *H* are the vertices of a triangle.

In Δ*EFH*, ∠ *HEF* is an_____________ angle. (Note: ∠ *HEF* is another name for ∠ *AEF*) and ∠ *HFG* is an _________ angle. (Note: ∠ *HFG* is another name for ∠ *CFG*)

We were given that m ∠ *HEF* = m __________________________.

Why is this a problem? ____

What conclusion can you draw? _______________________________________

2. Prove: If two lines are perpendicular to the same line, then they are parallel.

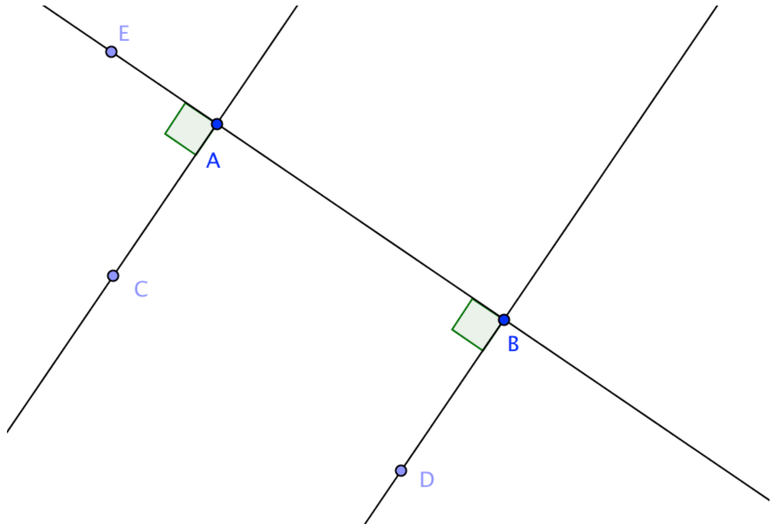
Complete this two-column proof.

Given:

$\overleftrightarrow{CA}$ is perpendicular to $\overleftrightarrow{AB}$
$\overleftrightarrow{DB}$ is perpendicular to $\overleftrightarrow{AB}$

Prove:

$\overleftrightarrow{CA}$ is parallel to $\overleftrightarrow{DB}$



Proof:

Statement	Reason
1. $\overleftrightarrow{CA}$ is perpendicular to $\overleftrightarrow{AB}$	1. ____________________
2. m $\angle$ *CAE* = 90°	2. ____________________
3. $\overleftrightarrow{DB}$ is perpendicular to $\overleftrightarrow{AB}$	3. ____________________
4. m $\angle$ *DBA* = 90°	4. ____________________
5. m $\angle$ *CAE* = m $\angle$ _________	5. ____________________
6. ________________________________	6. ____________________

Activity 3.3.3. Proving Lines are Parallel

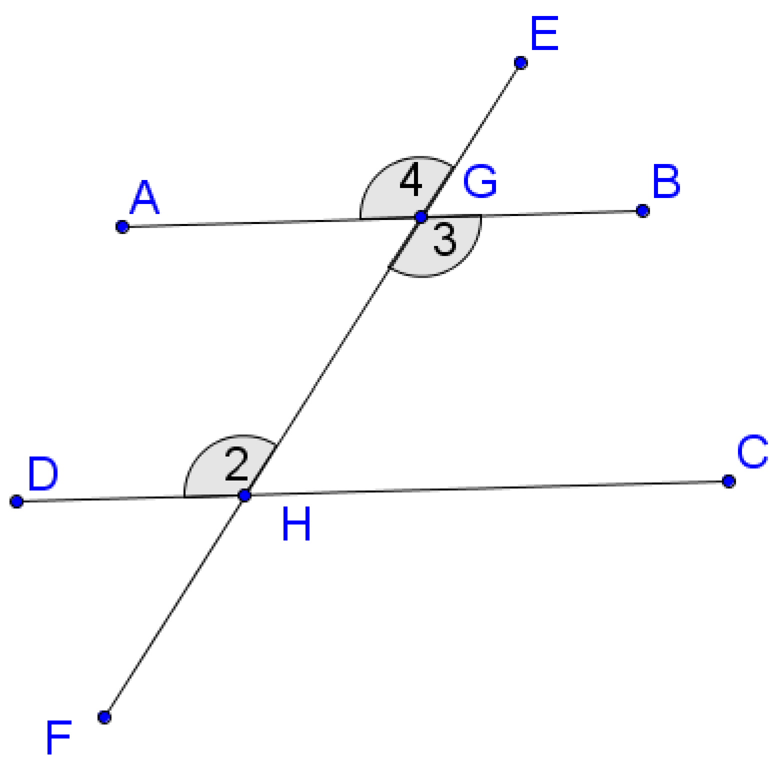
We have already proved (in activity 3.3.2) that if two lines cut by a transversal form congruent corresponding angles, then the two lines are parallel. We will now use this fact to prove the converses of two other theorems about parallel lines.

1. Parallel Lines Alternate Interior Angles Converse

Given: $m\angle 2 = m\angle 3$

Prove: $AB \parallel DC$

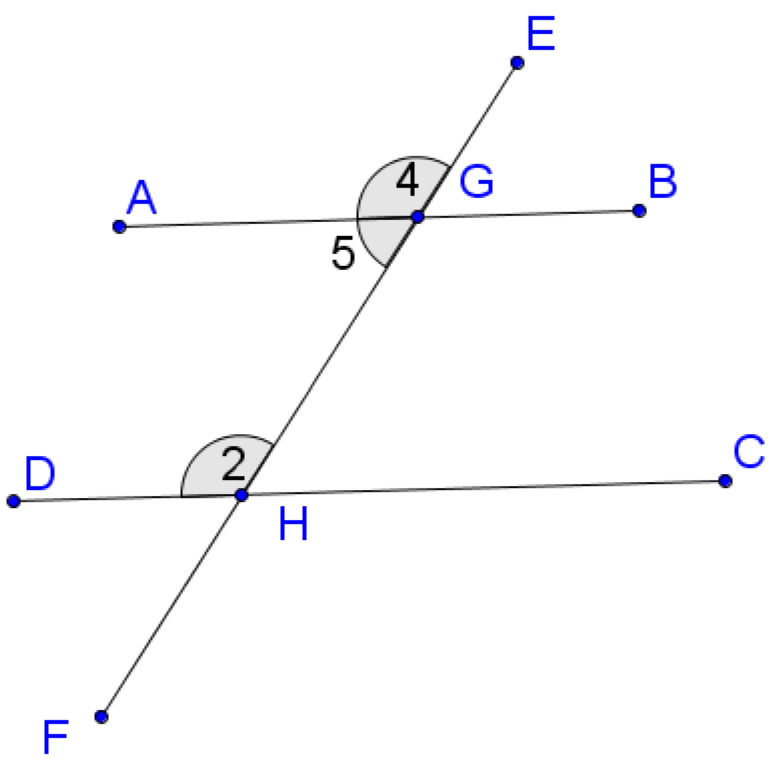
.



2. Parallel Lines Same Side Interior Angles Converse

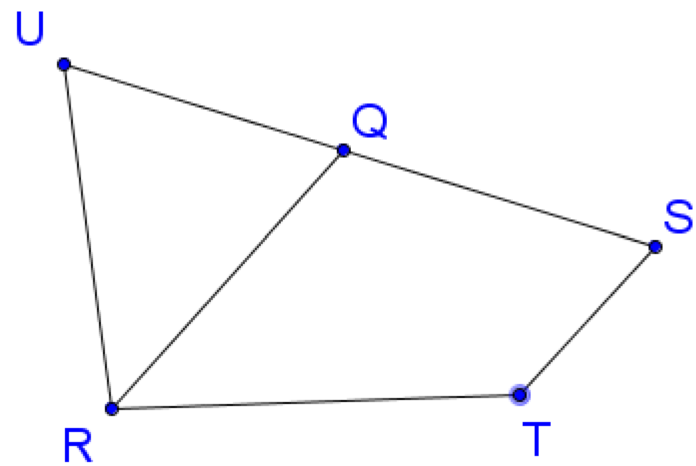
Given: ∠2 and ∠5 are supplementary

Prove: $AB \parallel DC$



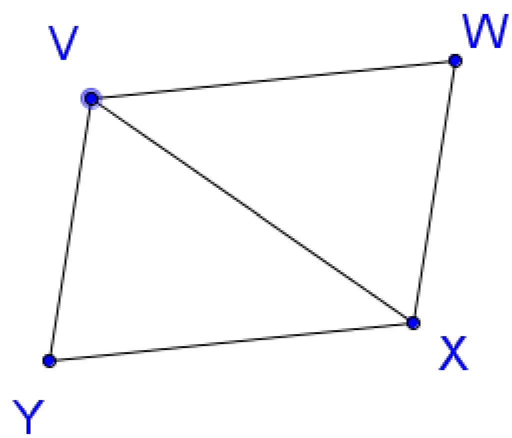
3. Given: $RU = RQ$ and $\angle RUQ \cong \angle TSQ$

   Prove: $RQ \parallel ST$



4. Given: $VY = WX$ and $VW = YX$

   Prove: $VY \parallel WX$



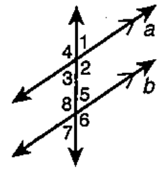
A

LESSON 3-3

Reading Strategies

Use a Graphic Organizer

Line *a* and line *b* are parallel. This can be proven in four different ways.



$a \parallel b$ if . . .

Converse of the Corresponding Angles Postulate
If $\angle 1 \cong \angle 5$, then $a \parallel b$.
If $\angle 2 \cong \angle 6$, then $a \parallel b$.
If $\angle 4 \cong \angle 8$, then $a \parallel b$.
If $\angle 3 \cong \angle 7$, then $a \parallel b$.

OR

Converse of the Alternate Interior Angles Theorem
If $\angle 2 \cong \angle 8$, then $a \parallel b$.
If $\angle 3 \cong \angle 5$, then $a \parallel b$.

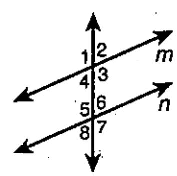
OR

Converse of the Alternate Exterior Angles Theorem
If $\angle 1 \cong \angle 7$, then $a \parallel b$.
If $\angle 4 \cong \angle 6$, then $a \parallel b$.

OR

Converse of the Same-Side Interior Angles Theorem
If $m\angle 2 + m\angle 5 = 180°$, then $a \parallel b$.
If $m\angle 3 + m\angle 8 = 180°$, then $a \parallel b$.

In Exercises 1–4, use the given information to determine the theorem or postulate that proves $m \parallel n$.



1. $\angle 1 \cong \angle 7$

____

2. $m\angle 4 + m\angle 5 = 180°$

____

3. $\angle 5 \cong \angle 3$

____

4. $\angle 8 \cong \angle 4$

____

5. If $m\angle 1 = 47°$ and $m\angle 5 = 49°$, are the lines parallel? Explain.

____

6. If $m\angle 3 = 119°$, what does the measure of $\angle 6$ need to be to prove $m \parallel n$?

____

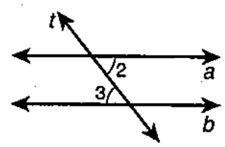
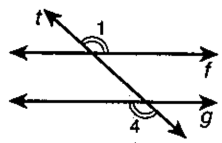
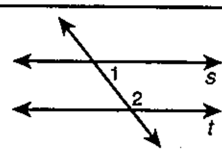
B

LESSON 3-3

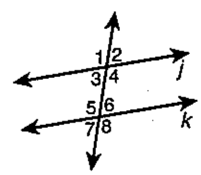
Reteach

Proving Lines Parallel continued

You can also prove that two lines are parallel by using the converse of any of the other theorems that you learned in Lesson 3-2.

Theorem	Hypothesis	Conclusion
Converse of the Alternate Interior Angles Theorem	 $\angle 2 \cong \angle 3$	$a \parallel b$
Converse of the Alternate Exterior Angles Theorem	 $\angle 1 \cong \angle 4$	$f \parallel g$
Converse of the Same-Side Interior Angles Theorem	 $m\angle 1 + m\angle 2 = 180°$	$s \parallel t$

For Exercises 3–5, use the theorems and the given information to show that $j \parallel k$.



3. Given: $\angle 4 \cong \angle 5$

4. Given: $m\angle 3 = 12x°$, $m\angle 5 = 18x°$, $x = 6$

5. Given: $m\angle 2 = 8x°$, $m\angle 7 = (7x + 9)°$, $x = 9$

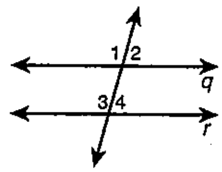
LESSON 3-3

Reteach

Proving Lines Parallel

Converse of the Corresponding Angles Postulate	If two coplanar lines are cut by a transversal so that a pair of corresponding angles are congruent, then the two lines are parallel.

You can use the Converse of the Corresponding Angles Postulate to show that two lines are parallel.



Given: $\angle 1 \cong \angle 3$

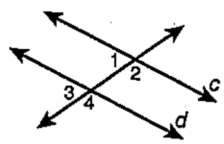
$\angle 1 \cong \angle 3$	$\angle 1 \cong \angle 3$ are corresponding angles.
$q \parallel r$	Converse of the Corresponding Angles Postulate

Given: $m\angle 2 = 3x°$, $m\angle 4 = (x + 50)°$, $x = 25$

$m\angle 2 = 3(25)° = 75°$	Substitute 25 for x.
$m\angle 4 = (25 + 50)° = 75°$	Substitute 25 for x.
$m\angle 2 = m\angle 4$	Transitive Property of Equality
$\angle 2 \cong \angle 4$	Definition of congruent angles
$q \parallel r$	Converse of the Corresponding Angles Postulate

For Exercises 1 and 2, use the Converse of the Corresponding Angles Postulate and the given information to show that *c* || *d*.

1. **Given:** $\angle 2 \cong \angle 4$



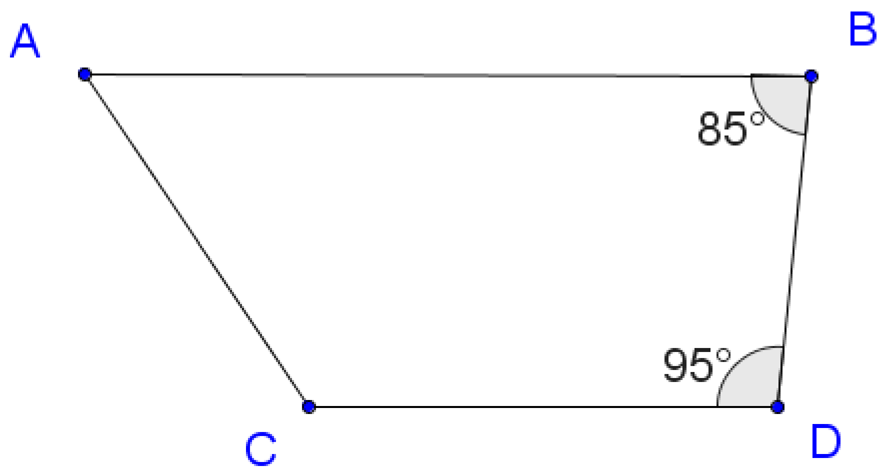
2. **Given:** $m\angle 1 = 2x°$, $m\angle 3 = (3x - 31)°$, $x = 31$

Activity 3.3.5 Proofs with Parallel Lines

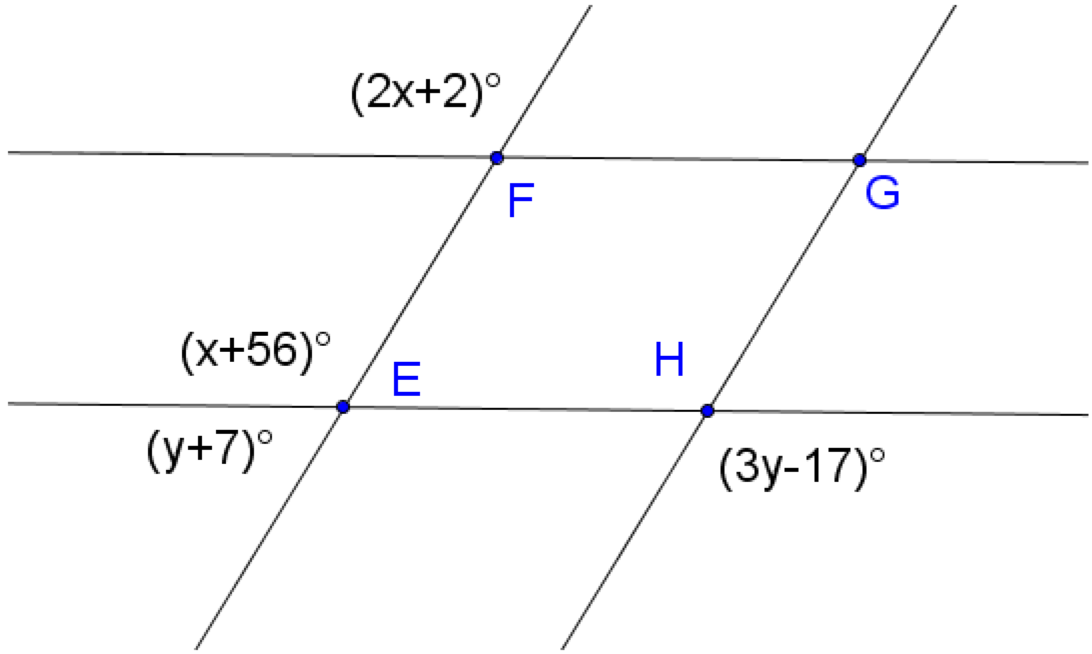
In each example, a figure is drawn with some information on the figure. Following will be some statements that can be either proved valid or disproved.

For this activity, the class should be divided into groups of 3 students. For each example, Student A will select a statement, Student B will try to prove or disprove the statement. Student C will offer suggestions to assist Student B. Once all of the statements in the example are completed, the students should switch roles. Each student will have an opportunity to be Student A, B and C.

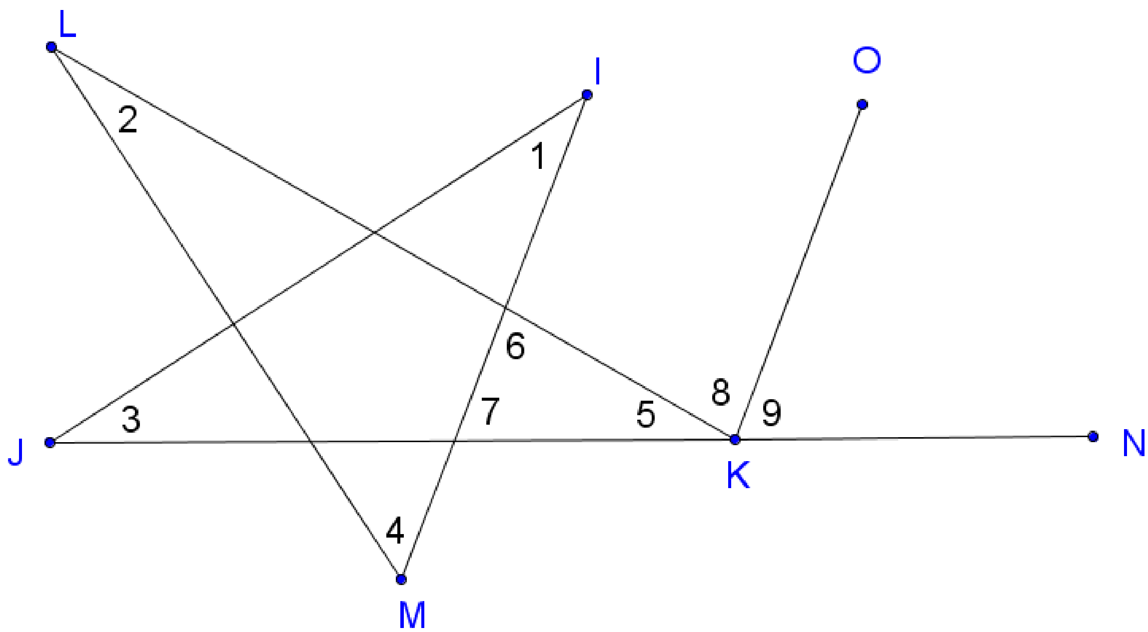
1. Use the figure below to prove or disprove the following statements
   a. $\overline{AB} \parallel \overline{DC}$
   b. $ABDC$ is a trapezoid
   c. Given that $m\angle C = 124°$, $ABDC$ is an isosceles trapezoid



2. Use the figure below to prove or disprove the following statements
   a. Is there a value for ***x*** such that $\overline{FG} \parallel \overline{EH}$? What is that value?
   b. Is there a value for ***y*** such that $\overline{EF} \parallel \overline{GH}$? What is that value?
   c. Can $\overline{FG} \parallel \overline{EH}$ and $\overline{EF} \parallel \overline{GH}$ at the same time?



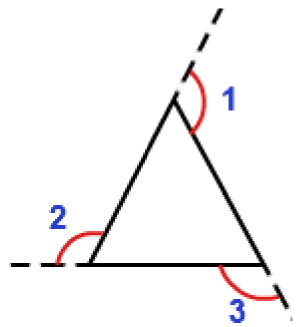
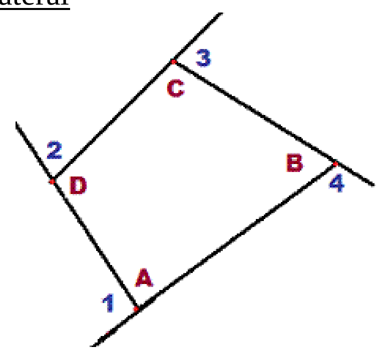
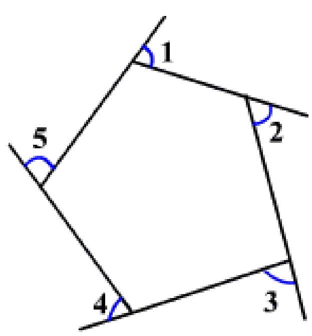
3. Use the figure below to answer the questions. It is given that $\overline{IM} \parallel \overline{OK}$
   a. How can $m\angle 6$ be re-written as in terms of other angles in the diagram?
   b. How can $m\angle 7$ be re-written as in terms of other angles in the diagram?
   c. How can $m\angle 8$ be re-written as in terms of other angles in the diagram?
   d. How can $m\angle 9$ be re-written as in terms of other angles in the diagram?
   e. What can this diagram be used to prove? (HINT: We have already proved this to be true in this course!!)



Activity 3.4.1 Exterior Angles of a Polygon

Sum of Exterior Angles of a Polygon Conjecture: The sum of the exterior angles (one at each vertex) for any convex polygon is _______________.

Measure the exterior angles (one at each vertex) for each of the convex polygons. Record your answers in the appropriate spaces below.

Triangle  Measure of Exterior Angle 1: ___________ Measure of Exterior Angle 2: ___________ Measure of Exterior Angle 3: ___________ Sum of Exterior Angles: _______________	Quadrilateral  Measure of Exterior Angle 1: ___________ Measure of Exterior Angle 2: ___________ Measure of Exterior Angle 3: ___________ Measure of Exterior Angle 4: ___________ Sum of Exterior Angles: _______________
Pentagon  Measure of Exterior Angle 1: ___________ Measure of Exterior Angle 2: ___________ Measure of Exterior Angle 3: ___________ Measure of Exterior Angle 4: ___________ Measure of Exterior Angle 5: ___________ Sum of Exterior Angles: _______________	Hexagon Draw a hexagon using your ruler. Measure the six exterior angles of your hexagon. Measure of Exterior Angle 1: ___________ Measure of Exterior Angle 2: ___________ Measure of Exterior Angle 3: ___________ Measure of Exterior Angle 4: ___________ Measure of Exterior Angle 5: ___________ Measure of Exterior Angle 6: ___________ Sum of Exterior Angles: _______________

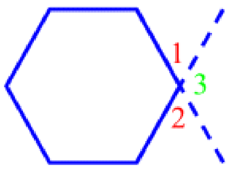
Compare your exterior angle sum with another student. Is their sum the same? Is this surprising?

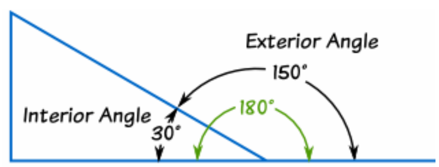
Your exterior angle sum: _______________

Student ___________ (name) exterior angle sum: _____________

What conclusions can you make? Do you think this works for any polygon? Write down some of your thoughts. Discuss this with your partner and write your combined thoughts in the space provided.

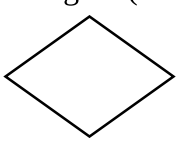
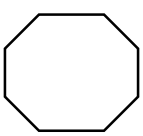
Notice: It is possible to draw two (equal) exterior angles at each vertex of a polygon. Either ∠1 or ∠2 below are exterior angles. ∠3 is not considered an exterior angle. The sum of the exterior angles formula uses only ONE exterior angle at each vertex. The exterior angle and the adjacent interior angle form a linear pair (are supplementary).





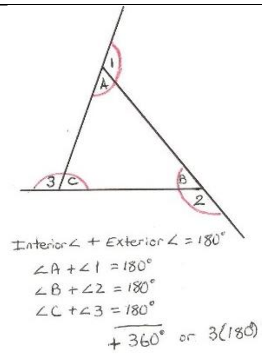
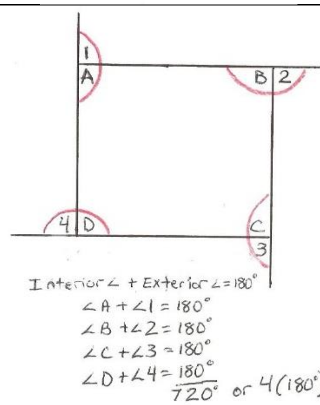
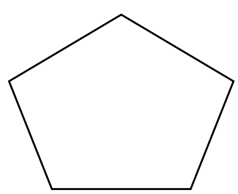
Review your conjecture:
Sum of Exterior Angles of a Polygon Conjecture: The sum of the exterior angles (one at each vertex) for any convex polygon is ________________.

What is the sum of the exterior angle measures, one at each vertex, of each polygon?

a) Pentagon (5-sided polygon) __________________
b) Heptagon (7-sided polygon) __________________
c) Decagon (10-sided polygon) __________________
d)  __________________
e)  __________________

At each vertex of a polygon there can be drawn an exterior angle, which is supplementary to the adjacent interior angle at the vertex.

Complete the table below:

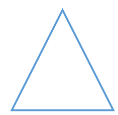
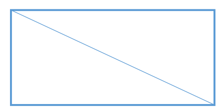
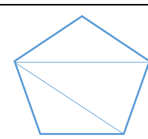
Convex Polygon	Number of vertices	Diagram of interior and exterior angle pairs at each vertex	Sum of all interior and exterior angle pairs at each vertex
Triangle	3		3(180°) or 540°
Quadrilateral			4(180°) or
Pentagon			5(180°) or
n-gon	n		*n*(180°)

The sum of all interior and exterior angle pairs at each vertex of a polygon is ________.

In Unit 3- Investigation 1, you learned about the sum of interior angles of polygons.

Triangle Sum Theorem: In any triangle the sum of the interior angles is 180°.
Quadrilateral Sum Theorem: In any convex quadrilateral the sum of the interior angles is 360°.
Polygon Sum Theorem: In any convex polygon with n sides, the sum of the interior angles is 180°(n – 2).

Complete the table below:

Convex Polygon	Number of sides	Diagram of interior angles	Sum of interior angles
Triangle			
Quadrilateral			(4–2)180° = 360°
Pentagon			
n-gon	*n*	(*n*–2) 180°	

Complete the Algebraic Proof:

The sum of all interior and exterior angle pairs at each vertex of a polygon is __________.

By the Polygon sum theorem, the sum of interior angles of a polygon is __________.

Next, take the sum of all the interior and exterior angle pairs at each vertex of a polygon and subtract the sum of interior angles of a polygon, to get the sum of exterior angles of a polygon.

Then the sum of the exterior angles is ________– __________.

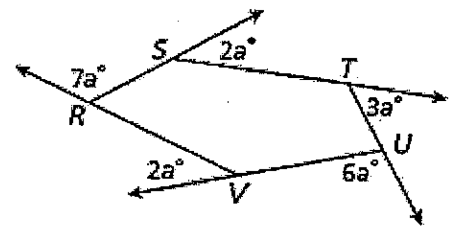
Simplified this is: 180°*n* – 180°*n* + 360° = ~~180°*n* – 180°*n*~~ + 360° = __________.

Therefore, the sum of exterior angles of a polygon is 360°.

Examples:

Find the measure of each exterior angle of a regular hexagon.

B Find the value of a in polygon $RSTUV$.



C **The measure of one exterior angle of a convex polygon is 20°. How many sides does the figure have? Name the polygon appropriately.**

Exercises

Find the sum of the exterior angle measure of each convex regular polygon.

1. octagon 2. decagon 3. heptagon 4. 102-gon

Find the measure of one exterior angle of each regular polygon.

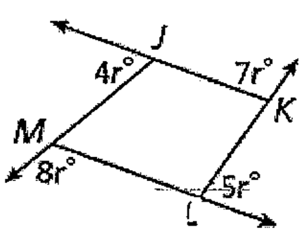
5. triangle 6. dodecagon 7. 20-gon 8. quadrilateral

The measure of one exterior angle of a regular convex polygon is given. How many sides does the figure have? Name the polygon appropriately.

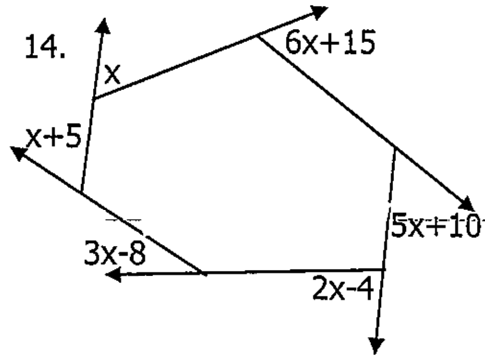
9. 72° 10. 45° 11. 30° 12. 120°

Find r.

13.

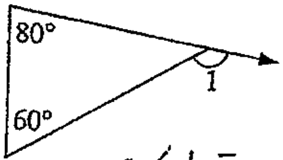


14.



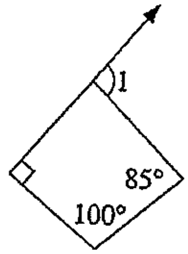
Find the measure of ∠1 in each figure.

13.



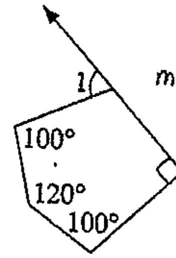
m∠1 = ______

14.



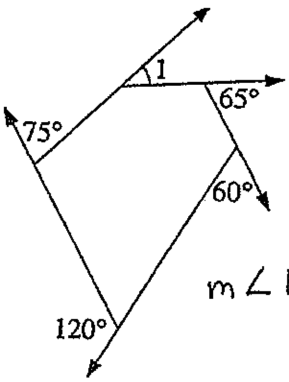
m∠1 = ______

15.



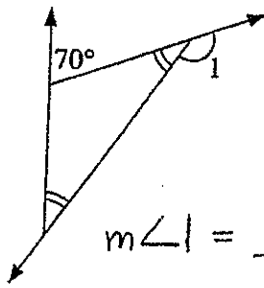
m∠1 = ______

16.



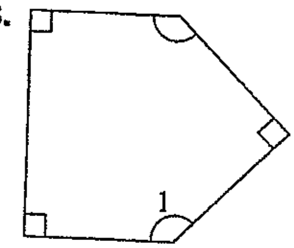
m∠1 = ______

17.



m∠1 = ______

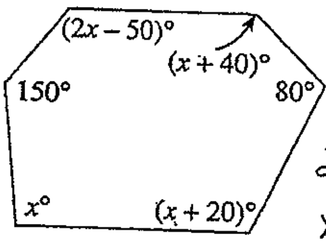
18.



m∠1 = ______

Find the measure of each angle.

19.



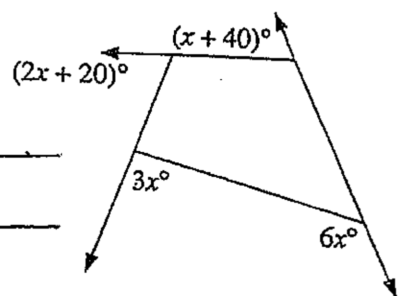
$2x-50=$ ______

$x+40=$ ______

$x+20=$ ______

$x=$ ______

20.



$x=$ ______

$x+40=$ ______

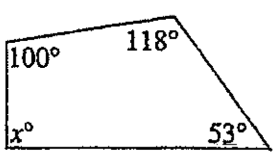
$2x+20=$ ______

$3x=$ ______

$6x=$ ______

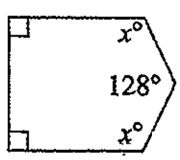
Find each unknown angle measure.

18.



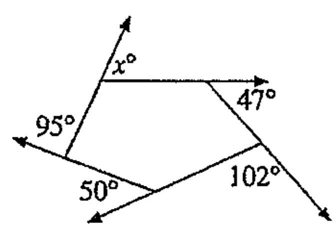
$x=$ ______

19.



$x=$ ______

20.



$x=$ ______

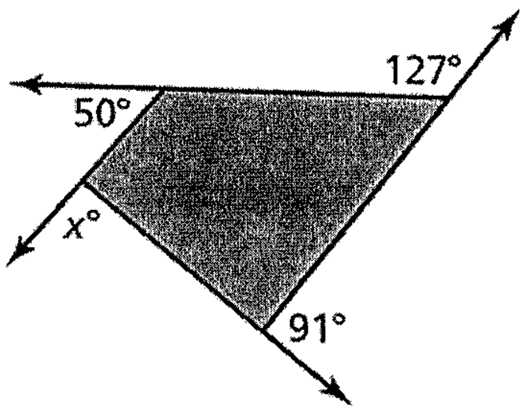
1) To calculate the **interior angle sum of a polygon**, you first subtract ____ from the number of _______ and then multiply this difference by _____°.

This formula can be written as (___ - ___)·180°.

2) The **sum of the exterior angles** of ALL polygons is _______°.

3) a) Calculate the value of x *(exterior angle).* _______

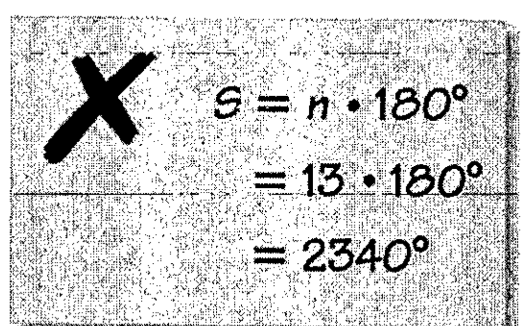
b) Calculate the measure of <u>each</u> *interior* angle of the quadrilateral. What is the <u>sum</u> of the *interior* angles? _________



**Reminder - The interior and exterior angle at each vertex are supplementary.

4) Can a pentagon have interior angles that measure 120°, 105°, 65°, 150°, and 95°? *Explain.*

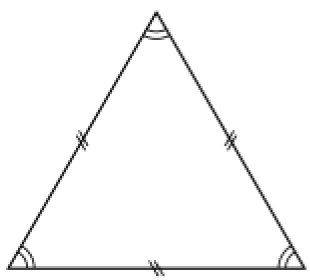
5) Describe and correct the error made below in finding the <u>sum</u> of the *interior* angle measures of a 13-gon

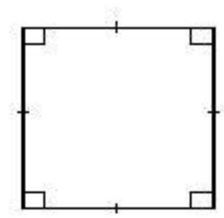


Activity 3.4.2 Angles in Regular Polygons

Angles in a Regular Polygon Conjecture: In a regular polygon the measure of each interior angle is __________

Below are an equilateral triangle and a square. Find the measure of each angle in these regular polygons.





Sum of interior angles:
Measure of each interior angle:

Sum of interior angles:
Measure of each interior angle:

What do you notice?

Recall that the sum of all interior angle measures in a regular polygon with *n* sides is $(n-2)180°$. Also, notice that all of the interior angles in a regular polygon have the same measure. Then we could find the measure of each interior angle by taking the interior angle sum and dividing by the number angles: $\frac{(n-2)180°}{n}$.

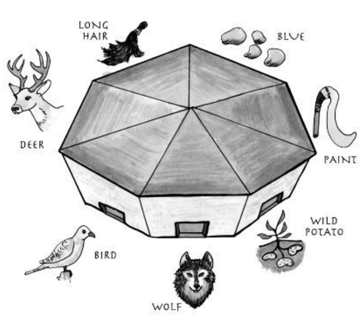
Similarly, recall that the sum of all exterior angle measures of a regular polygon with *n* sides is 360°. Then we could find the measure of each exterior angle by taking the exterior angle sum and dividing by the number angles: $\frac{360°}{n}$.

Complete the table below:

Polygon	Number of sides n	Interior angle sum	Measure of each interior angle	Measure of each exterior angle
Triangle				
Quadrilateral				
Pentagon				
Hexagon				
n-gon				

Angles in a Regular Polygon Theorem: In a regular polygon the measure of each interior angle is $\frac{(n-2)180°}{n}$ and the measure of each exterior angle is $\frac{360°}{n}$.

Practice Problem:
The Cherokee People originally inhabited the southern Appalachian mountain region of North Carolina, Tennessee, and Georgia. Their society was organized into 7 clans. Their council houses had seven sections, one for each clan to sit in. The council house was built in the shape of a regular heptagon. Find the measure of each interior angle and the measure of each exterior angle.

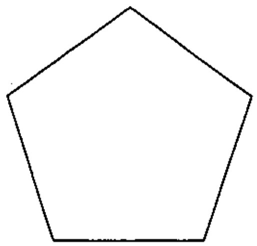


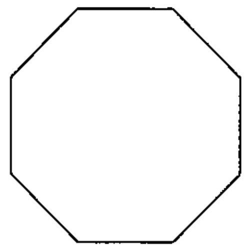
Geometry Polygons

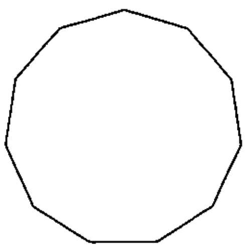
Sum of the interior angles of a polygon	$(n-2)180$
Sum of the exterior angles of a polygon	$360°$
Each interior angle of a regular polygon	$\frac{(n-2)180}{n}$
Each exterior angle of a regular polygon	$\frac{360}{n}$

Polygons and Angles

Find the measure of one interior angle in each polygon. Round your answer to the nearest tenth if necessary.

1)


2)


3)


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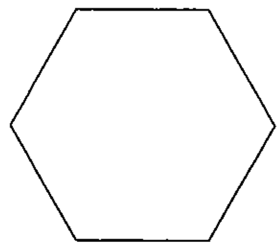

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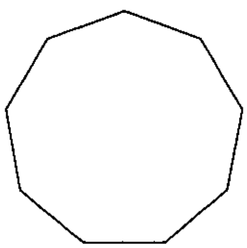

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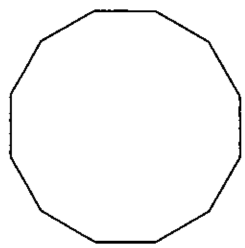


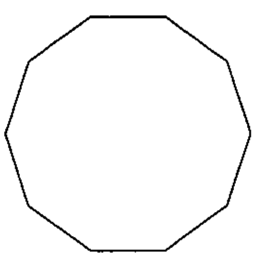
7) regular 24-gon

8) regular quadrilateral

9) regular 23-gon

10) regular 16-gon

Find the measure of one exterior angle in each polygon. Round your answer to the nearest tenth if necessary.

11)


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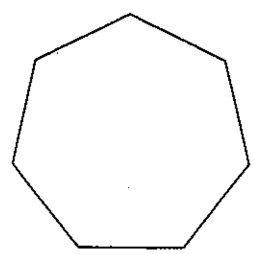

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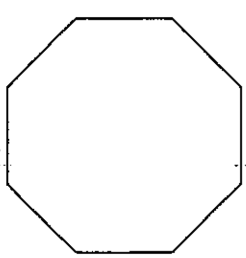

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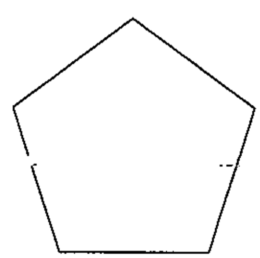

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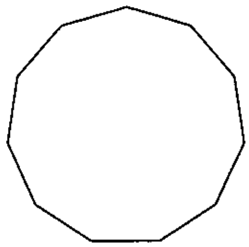

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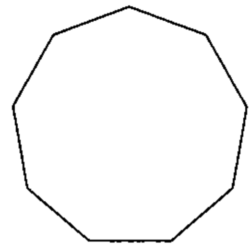


17) regular 13-gon

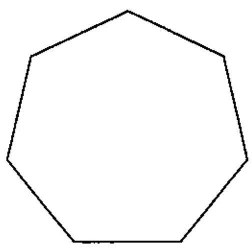
18) regular 16-gon

19) regular 20-gon

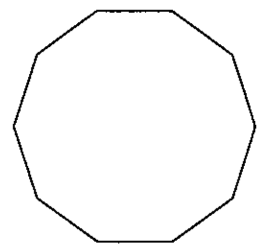
20) regular 23-gon

Find the interior angle sum for each polygon. Round your answer to the nearest tenth if necessary.

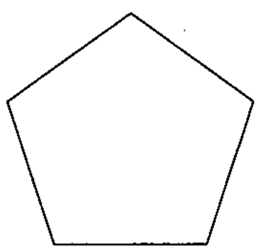
21)



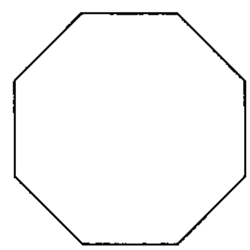
22)



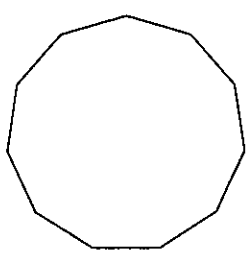
23)



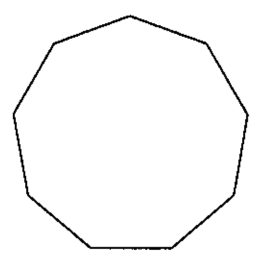
24)



25)



26)



27) regular quadrilateral

28) regular 18-gon

29) regular dodecagon

30) regular 15-gon

Critical thinking questions:

31) What is the exterior angle sum of a 500-gon?

32) Is there a regular polygon with an interior angle sum of 9000°? If so, what is it?

Activity 3.4.3 Applying Angle Properties in Regular Polygons

Angles in a Regular Polygon Theorem: In a regular polygon the measure of each interior angle is $\frac{(n-2)180°}{n}$ and the measure of each exterior angle is $\frac{360°}{n}$.

Practice Problems:

1. A trampoline is in the shape of a regular octagon. What is the measure of each interior angle and each exterior angle?

2. The sum of interior angles in a regular polygon is 1080°. How many sides are in this regular polygon?

3. How many sides does a regular polygon have, if its interior angles equal 144°?

4. A dodecagon is a 12-sided polygon. In a regular dodecagon, what is the measure of each interior angle and each exterior angle?

5. The Santiago family is building a gazebo in their backyard as pictured below. The gazebo is in the shape of a regular hexagon. At each interior angle there will be a metal bracket, how many degrees is the does each metal bracket need to be?

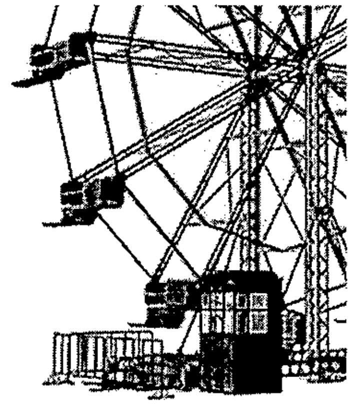
6. Each interior angle of a regular polygon measures 135°. How many sides does the polygon have?

7. If each exterior angle measures 40°, how many sides does this polygon have?

8. If each exterior angle measures 50°, how many sides does this polygon have? If the polygon is not possible, then explain why.

3.4.3 A

Geometry - *Polygons*



1) The car at each vertex of a Ferris wheel holds a maximum of 5 people. The sum of the interior angle measures of the Ferris wheel (a regular polygon) is 7740°.

   What is the maximum number of people the Ferris wheel can hold? ________

2) Do #26 on page 387. **Make sure to draw a diagram, write and solve an equation for 'a', and calculate ALL unknown angle measures (interior and exterior!)*

3) Do #42 on page 387. **Make sure to show work for <u>both</u> the number of sides AND the measure of each interior angle.*

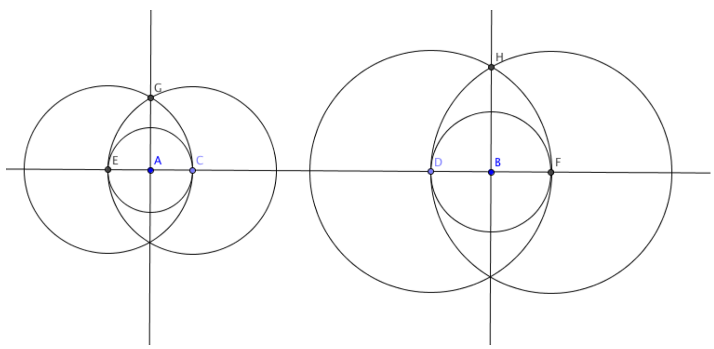
Activity 3.4.4 Constructing Regular Polygons with Compass and Straightedge

Some regular polygons may be constructed with compass and straightedge. Use these tools or software to perform these constructions. If you use software be sure to use only the straight object and circle tools.

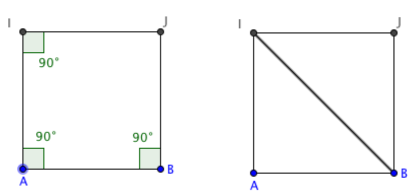
1. **Regular Triangle** (also known as an equilateral triangle). You already did this in Activity 2.6.2 and wrote a proof in Activity 2.7.1.

2. **Regular Quadrilateral** (also known as a square). You may have done this in Activity 2.6.3. If not, let's try it here.
   a. Start with a line through two points *A* and *B*. $\overline{AB}$ will be one side of the square

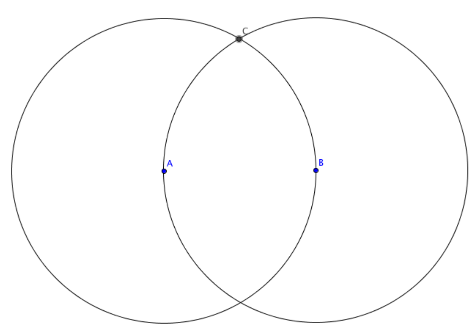
   b. Construct lines $\overleftrightarrow{AG}$ and $\overleftrightarrow{BH}$ perpendicular to $\overleftrightarrow{AB}$. You did this in Activity 2.7.3. You may want to review the activity or study the figure below.



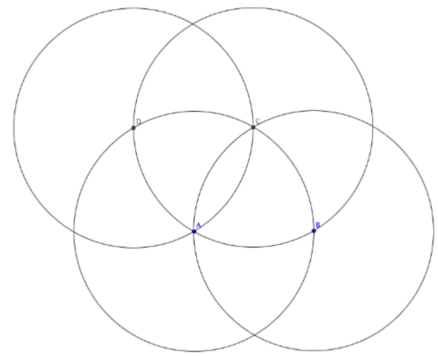
   c. Locate point *I* on ray $\overrightarrow{AG}$ so that $AI = AB$. Construct a line through *I* perpendicular to $\overrightarrow{AG}$ and let this line intersect $\overleftrightarrow{BH}$ at point *J*. In the space below, prove that quadrilateral *ABJI* is a square. (Hint: First show that m $\angle BJI = 90°$. Then show that $\Delta IAB \cong \Delta IJB$.)



3. **Regular Pentagon**. This construction uses the Golden Ratio and will be presented in Unit 8 Investigation 3.



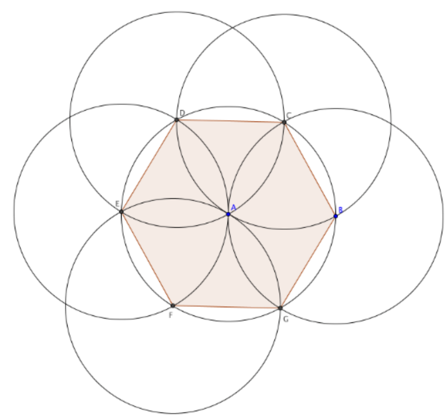
4. **Regular Hexagon**.
   a. Start with points *A* and *B*. Draw a circle with center *A* passing through *B* and a circle with center *B* passing through *A*. Label one of the points where the two circles intersect point *C*.



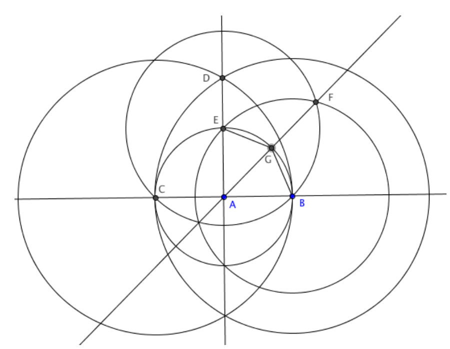
   b. Draw a circle with center *C* passing through *A*. Label *D* the point where this circle intersects circle *A*.

   c. Continue the process with a circle centered at *D* passing through *A* to locate point *E*. And then a circle with center at *E* and another one with center at *F*.

   d. You should end up with 6 points on circle *A* evenly spaced. Join *B* to *C* to *D* to *E* to *F* to *G* to *B* to form a hexagon.



   e. In the space below, prove that hexagon *BCDEFG* is regular. First show that all six sides are congruent and then show that all 6 angles measure 120°



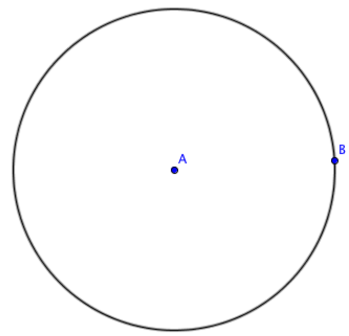
5. **Regular Heptagon.** Sorry, it is impossible to construct a regular polygon with 7 sides using only compass and straightedge!

6. **Regular Octagon.** If you like an extra challenge, try this one. Hint: The figure at the right shows how you might get started.

Activity 3.4.5 Constructing Regular Polygons with Other Tools

In the previous activity you were limited to using the Euclidean tools—compass and straightedge. Now we will allow other tools such as protractors, rulers, and the transformation tools of Geogebra or Geometer's Sketchpad.

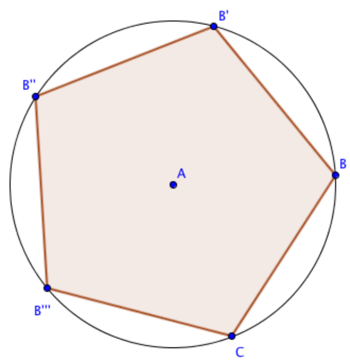
1. **Construct a regular polygon with any number of sides (*n*) inscribed in a given circle.**



   a. First divide 360° by the number of sides. For example if *n* = 5, $\frac{360°}{n} = \frac{360°}{5} = 72°$.

   b. Draw a circle with center *A* and radius $\overline{AB}$.

   c. Rotate *B* about point *A* with angle $\frac{360°}{n}$. In this case the angle is 72°.



   d. Rotate *B'* about point *A* with the same angle.

   e. Continue rotating until you have *n* equally spaced points on the circle.

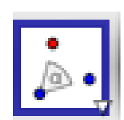
   f. Join the points on the circle to form a polygon.

   g. Measure the sides and the interior angles to verify that your polygon is regular.
   Sides = ____________ Angles = ___________

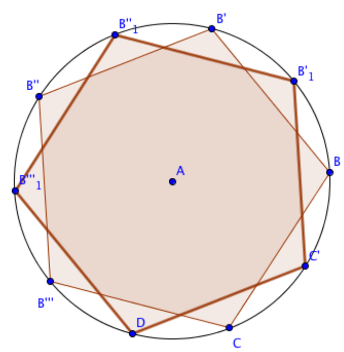
   h. Now try this with *n* = 10, *n* = 9, and *n* = 7.

2. **Explore the rotational symmetry of a regular polygon.**

   a. Start with the regular pentagon inscribed in a circle as shown above. Use the rotate tool.



   b. Select the pentagon, point A, and 36° for the angle of rotation. Observe that the rotated pentagon does not coincide with the original pentagon as shown.



   c. Undo the rotation so that only the original pentagon is showing.

   d. Now rotate the pentagon again about point A this time with 72° for the angle of rotation. What happens?

e. Rotate the pentagon 144°. Observe what happens.

f. Find one more angle of rotation that will map the pentagon onto itself.

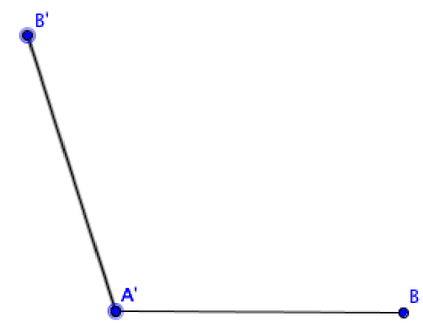
g. Make a conjecture about which angles of rotation will map the pentagon onto itself.

h. Suppose you start with a regular nonagon (n = 9), which you constructed in question 1h. What angle(s) of rotation will map the nonagon onto itself? Test your conjecture.

3. Constructing a regular polygon given one side. Again suppose n = 5, so we will be constructing a regular pentagon.

a. Start with a line segment $\overline{AB}$.

b. Find the measure of an interior angle of your regular polygon. Recall how to do this from Activity 3.4.2.



Show your calculation here:

c. Rotate segment $\overline{AB}$ about point A through the angle found in step (b).

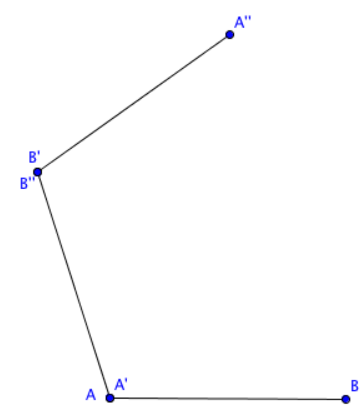
d. Now rotate segment $\overline{A'B'}$ about point B' through the same angle.

e. Continue the process until the regular polygon is completely formed.

f. Measure the sides and angles of the figure you have created to verify that this polygon is indeed regular.

Sides = __________ Angle measures = ___________

g. Repeat this process for some other value of n of your choosing.

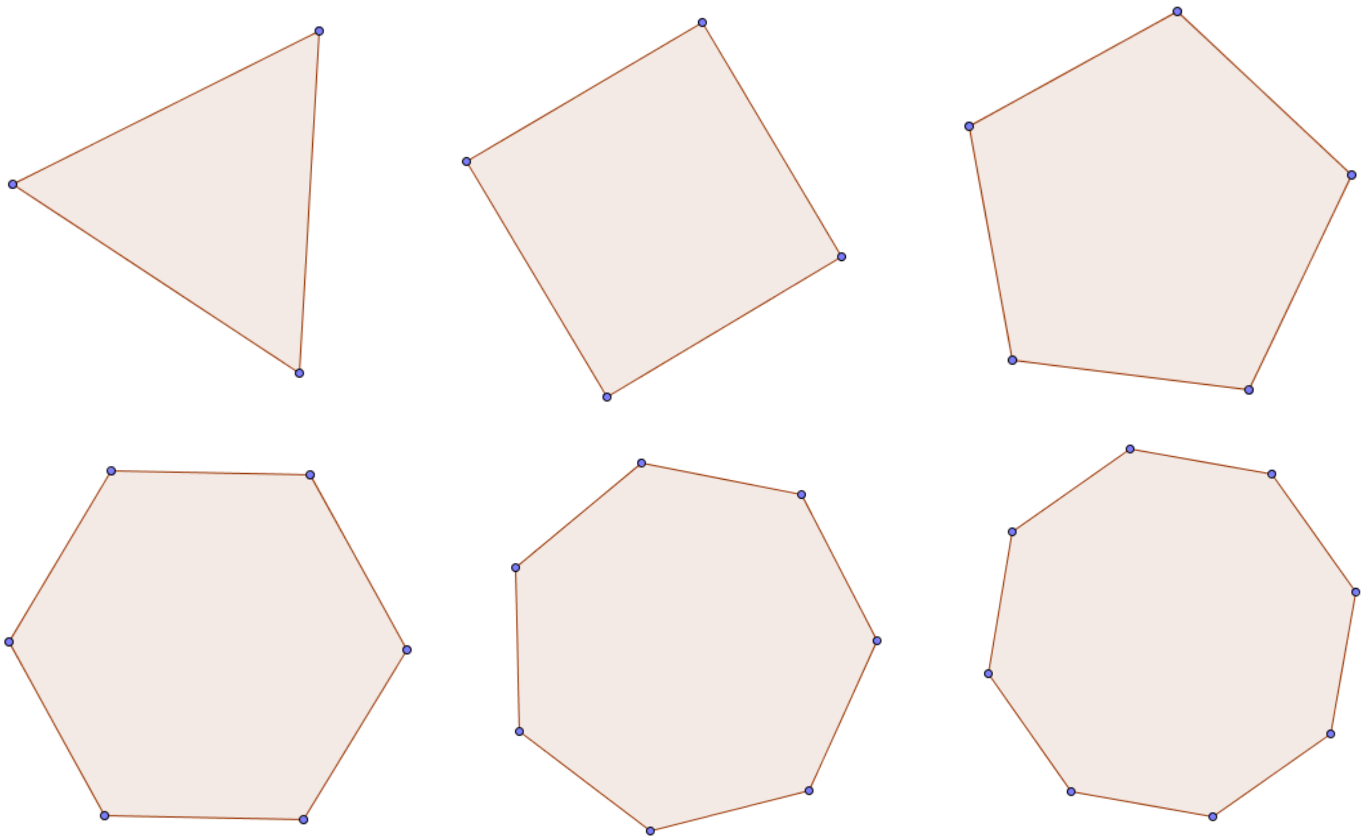


Activity 3.4.6 Lines of Symmetry in Regular Polygons

Use the template that comes with this activity.

1. Cut out the six regular polygons and fold along their lines of symmetry. You may want to work with a partner, each taking three polygons.

2. Sketch the lines of symmetry you found on the figures below.



3. Make conjectures about how many lines of symmetry a polygon with *n* sides has and where those lines are located. Also record any other patterns you notice.

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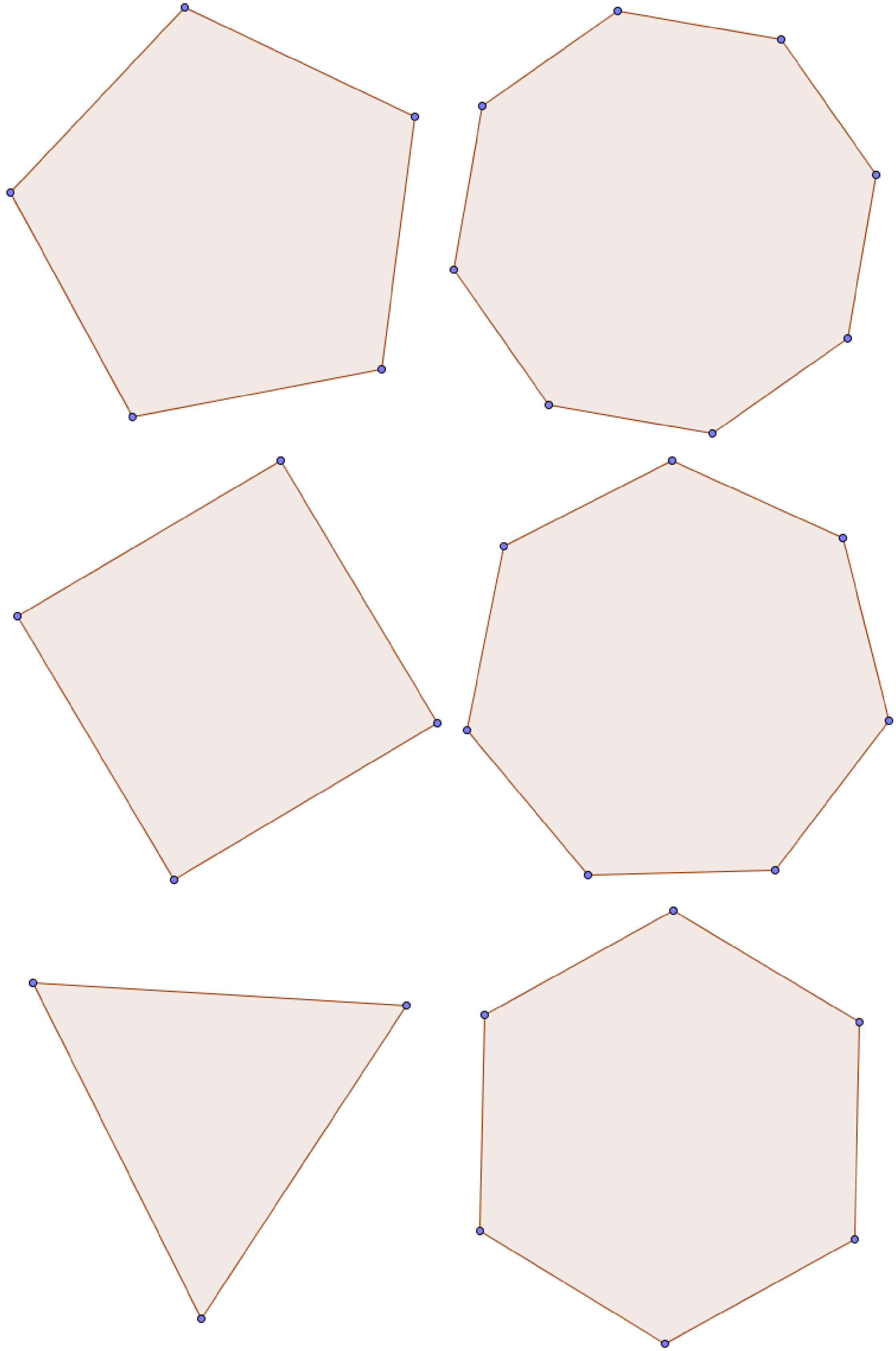

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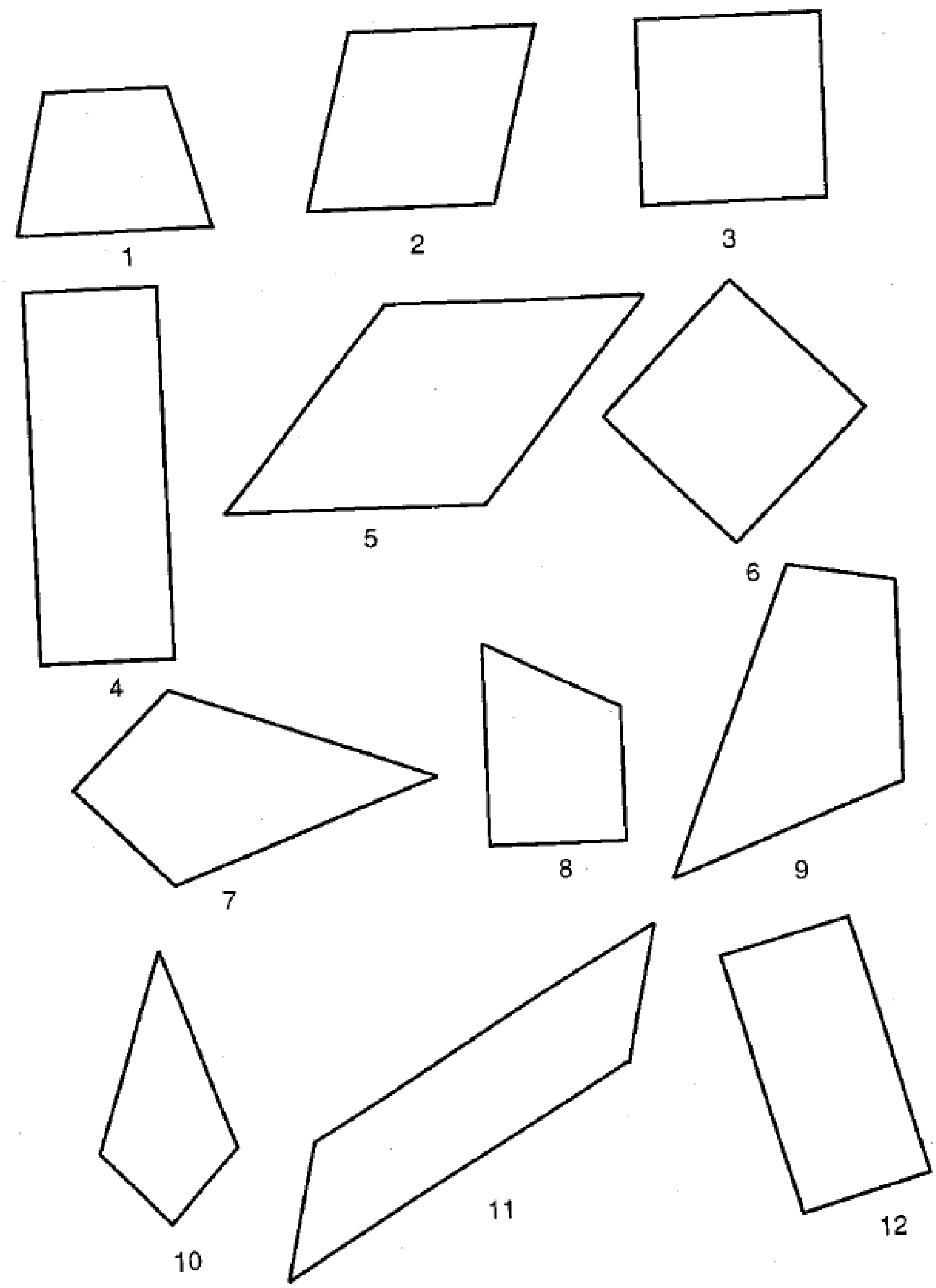
Activity 3.5.1 Classifying Quadrilaterals

Use these 12 quadrilaterals for this activity. You may assume that things are the way they appear. For example, if two sides appear to be equal you may assume that they are. If two sides appear to be parallel you may assume that they are; etc. You may also use rulers or protractors to check.



Figures from Craine and Rubenstein, 1993.

Answer the questions on the next page of this activity.

1. Which quadrilaterals have at least one pair of parallel sides?

2. Which quadrilaterals have two pairs of parallel sides?

3. Which quadrilaterals have at least two pairs of congruent adjacent sides?

4. Which quadrilaterals have at least two pairs of congruent adjacent angles?

5. Which quadrilaterals have four congruent sides?

6. Which quadrilaterals have four congruent angles?

7. Which quadrilaterals appeared in every answer to questions 1 through 6? What are these quadrilaterals called?

8. In this course we will use this definition of trapezoid: *A trapezoid is a quadrilateral with at least one pair of parallel sides.* Using this definition, which of the quadrilaterals are trapezoids?

9. In this course we will use this definition of isosceles trapezoid: *An isosceles trapezoid is a quadrilateral with at least two pairs of congruent adjacent angles.* Using this definition, which of the quadrilaterals are isosceles trapezoids?

10. *A parallelogram is a quadrilateral with two pairs of parallel sides*. Which of the quadrilaterals are parallelograms?

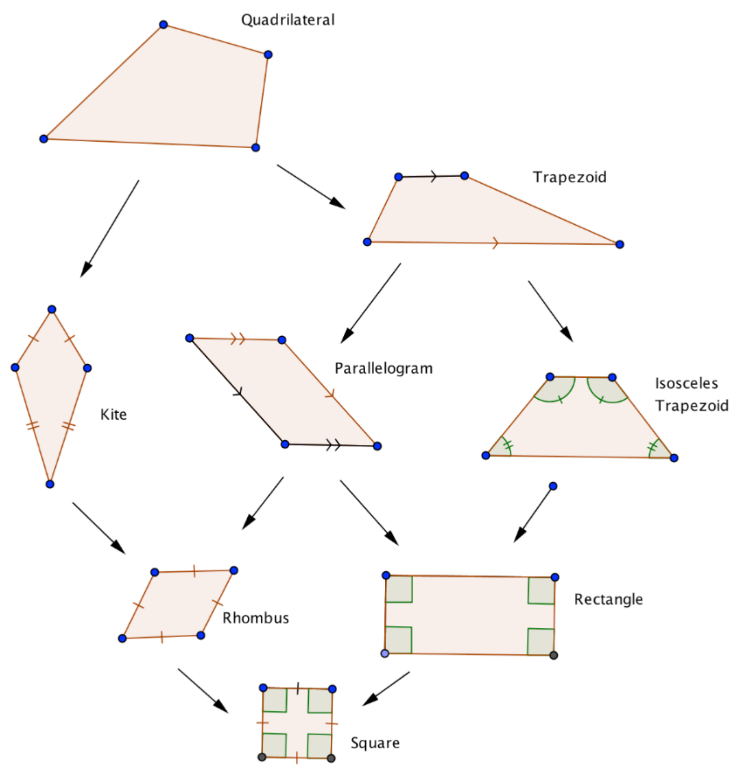
11. *A kite is a quadrilateral with two at least two pairs of congruent adjacent sides.* Which of the quadrilaterals are kites?

12. *A rectangle is a quadrilateral with four congruent angles*. Which of the quadrilaterals are rectangles?

13. *A rhombus is a quadrilateral with four congruent sides*. Which of the quadrilaterals rhombuses?

The Hierarchy of Quadrilaterals

The chart at the right shows the hierarchy of quadrilaterals. As you go down the chart each type of quadrilateral has more properties than the one above it.



14-19. Fill in the blanks:

14. If a quadrilateral is both a rectangle and a rhombus, then it is a ___________.

15. If a quadrilateral is both a kite and a parallelogram, then it is a ___________.

16. If a quadrilateral is both a parallelogram and an isosceles trapezoid, then it is a ___________.

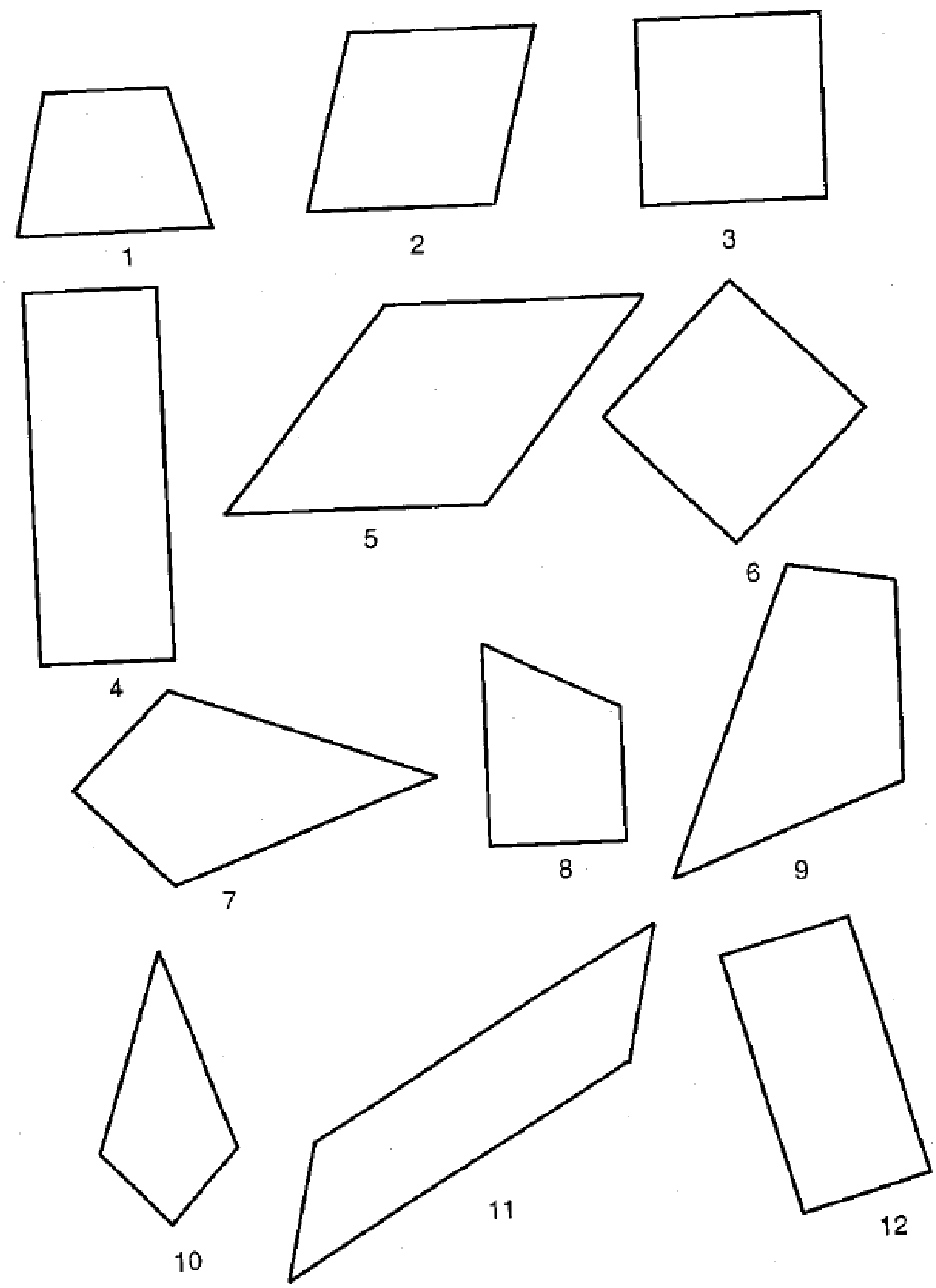
17. Every isosceles trapezoid is also a ___________ and a ___________.

18. Every parallelogram is also a ___________ and a ___________.

19. Every square is also a ______________, a ______________, a ______________, a ______________, a ______________, a ______________, and a ______________.

Activity 3.5.2a Diagonals of Quadrilaterals

Use these 12 quadrilaterals for this activity (same as Activity 3.5.1). Write the most specific name of each inside the quadrilateral. Your teacher will assign you to a group with one or two of the following quadrilaterals: Quadrilateral, Trapezoid, Isosceles Trapezoid, Parallelogram, Kite, Rhombus, Rectangle, and Square. My group's assigned quadrilateral is ___________________. Cut out the quadrilateral(s) that are labeled with your group name. Draw both diagonals with a ruler.



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1. Investigate which of the following properties are always true about your quadrilateral(s). Place a check mark in the appropriate box if the quadrilateral has the property.

	Congruent Diagonals	Perpendicular Diagonals	Diagonals Bisect Each Other
Quadrilateral			
Trapezoid			
Isosceles Trapezoid			
Kite			
Parallelogram			
Rhombus			
Rectangle			
Square			

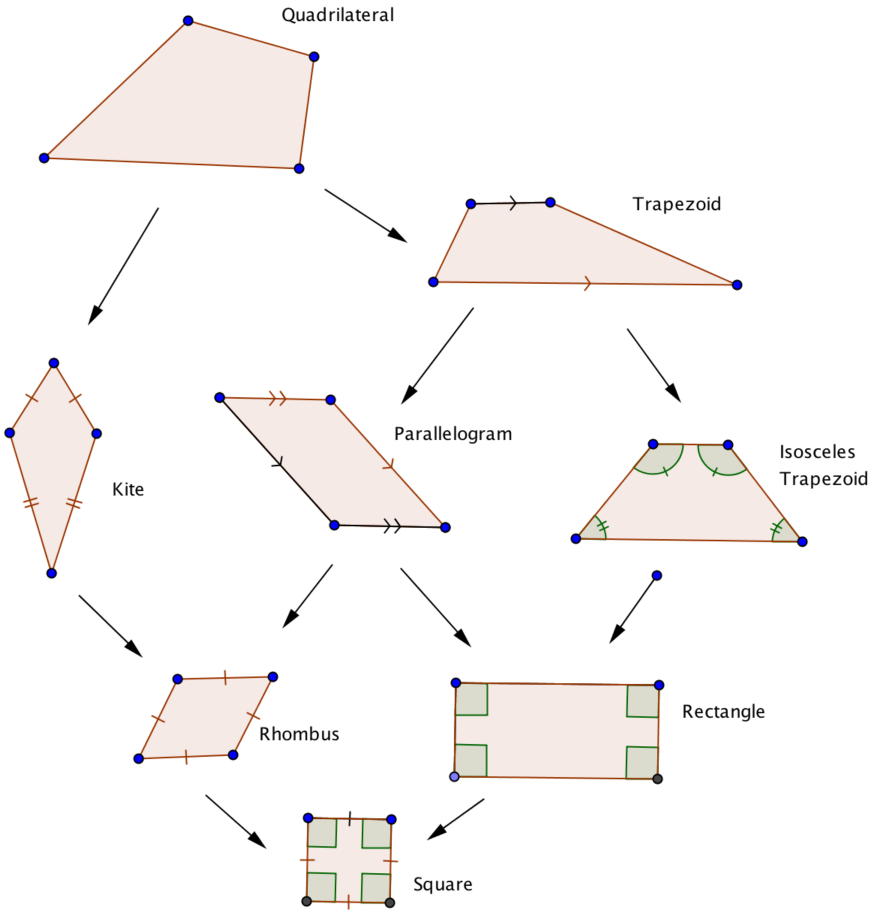
2. Assign one person from your group to be the rotator. When your teacher says to rotate, the rotator from each group will move to another group. During this time, explain what you learned with the new group member(s). Fill in the chart with the information that you learned. You will keep rotating until your chart is completed. Then return to your original group.

3. Which quadrilaterals have congruent diagonals?

4. Which quadrilaterals have perpendicular diagonals?

5. Which quadrilaterals have diagonals that bisect each other?

6. For each quadrilateral in the hierarchy, draw the diagonals and include any and all congruent or perpendicular markings and write the properties below. Remember that the quadrilaterals at the bottom of the hierarchy should have more properties than the quadrilaterals above them.



7. Answer the following with "Always," "Sometimes," or "Never."

a. If a quadrilateral is a rhombus, then the diagonals are ____________________ perpendicular.

b. If a quadrilateral is a parallelogram, then the diagonals are ________________ congruent.

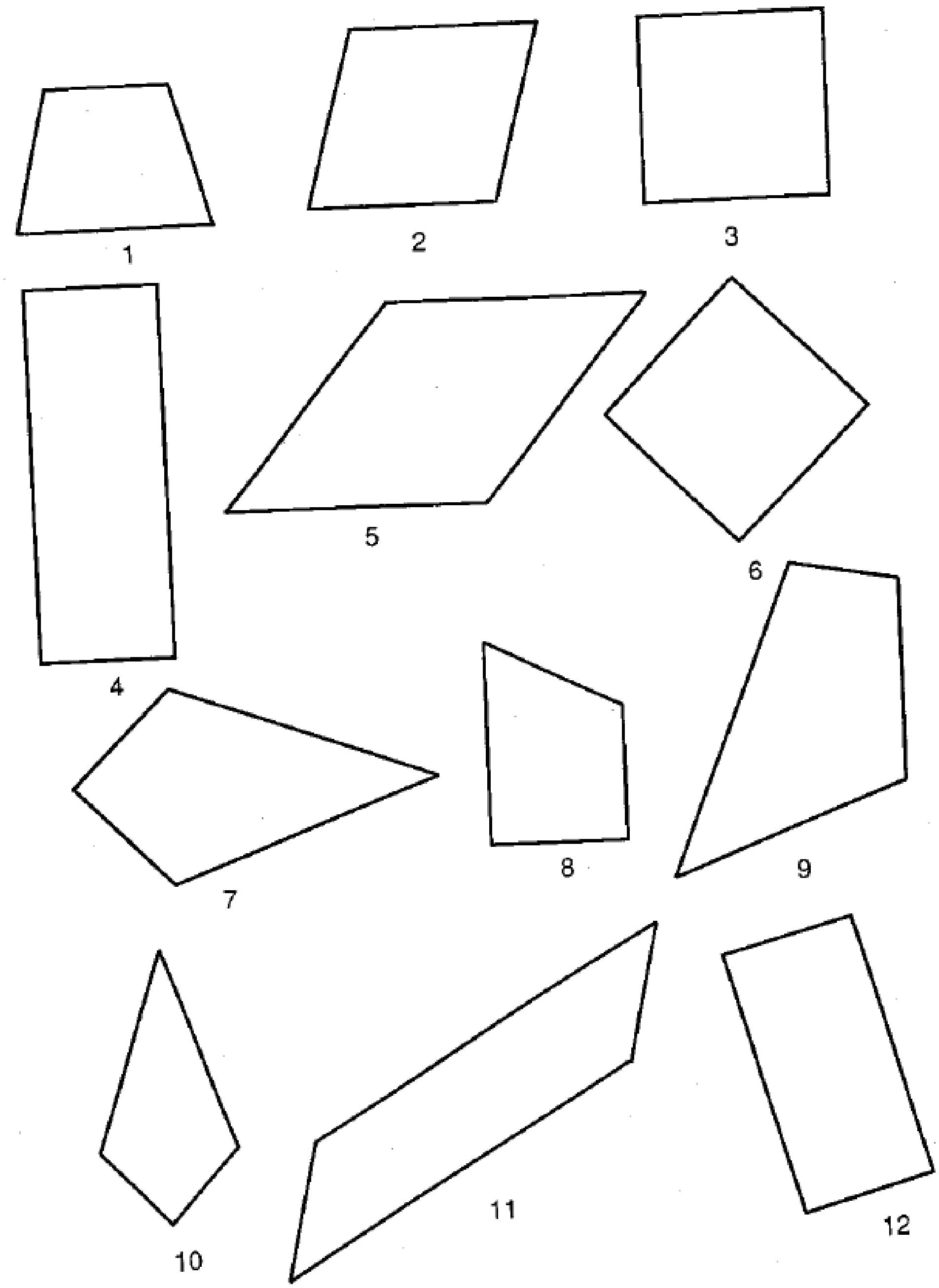
c. If a quadrilateral is a kite, then the diagonals __________________ bisect each other.

d. If a quadrilateral is an isosceles trapezoid, then the diagonals are ______________ congruent.

e. If a quadrilateral is rhombus, then the diagonals are _____________ perpendicular bisectors of each other.

Activity 3.5.3 Symmetry in Quadrilaterals

1. Your teacher will assign you to one or two of the quadrilaterals. Cut out the corresponding quadrilaterals below for your group.



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2. Investigate your quadrilateral(s) by filling in the following table. Use paper folding to help you.

	Sketch with lines of symmetry.	Degrees of Rotational Symmetry	Do the diagonals always bisect interior angles?
Quadrilateral			
Trapezoid			
Isosceles Trapezoid			
Kite			
Parallelogram			
Rhombus			
Rectangle			
Square			

3. Assign one person from your group to be the rotator. When your teacher says to rotate, the rotator from each group will move to another group. During this time, explain what you learned with the new group member(s). Fill in the chart on the previous page with the information that you learned. You will keep rotating until your chart is completed. Then return to your original group.

4. Which quadrilaterals have diagonals that bisect the interior angles?

5. Which quadrilaterals have exactly one line of symmetry?

6. Which quadrilaterals have two lines of symmetry?

7. Which quadrilaterals have four lines of symmetry?

8. Which quadrilaterals have 180 degree rotational symmetry?

9. Which quadrilaterals have 90 degree rotational symmetry?

10. Complete the following statements with “True” or “False.”

a. A parallelogram’s diagonals are always lines of symmetry. _____________

b. The diagonals of a rectangle bisect the interior angles. ______________

c. An isosceles trapezoid’s line of symmetry is the perpendicular bisector of the bases. ________

d. A kite always has two lines of symmetry. __________

e. A rhombus’s diagonals are always lines of symmetry. ______________

Activity 3.5.4 Properties of Parallelograms

You have been investigating properties of the different quadrilaterals. In this activity we will prove some properties of parallelograms.

First let's review the definition and properties that we investigated already.

1. A parallelogram is defined as a quadrilateral with two pairs of ________________ sides.

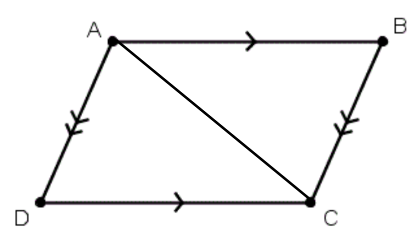
2. Earlier in this investigation you may have discovered these properties:
   a. The opposite sides of a parallelogram are ____________________.
   b. The opposite angles of a parallelogram are ________________
   c. The diagonals of a parallelogram ______________ each other.

Now, let's prove the properties as they follow from the definition of parallelogram.

3. **Parallelogram Opposite Sides Theorem**: If a quadrilateral is a parallelogram, then its opposite sides are congruent.

Fill in the blanks to complete the proof. Mark pairs of congruent sides and angle on the diagram.

Given: *ABCD* is a parallelogram.
Prove: $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$
Draw diagonal $\overline{AC}$



Since ABCD is a parallelogram, $\overline{AB} \parallel \overline{CD}$ and ____________.

Since $\overline{AB} \parallel \overline{CD}$, $\angle BAC \cong$ ________ because Alternate Interior Angles formed by parallel lines are congruent.

Since ____ || _____, $\angle BCA \cong$ _______because ____.

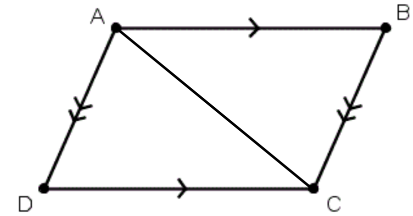
We also know that ______ $\cong \overline{AC}$ because $\overline{AC}$ is a shared side.

$\Delta ABC \cong \Delta$_____ by the ASA Congruence Theorem.

Therefore, $\overline{AB} \cong \overline{CD}$ and ______ $\cong$ ________ because CPCTC.

4. **Parallelogram Opposite Angles Theorem**: If a quadrilateral is a parallelogram, then its opposite angles are congruent.

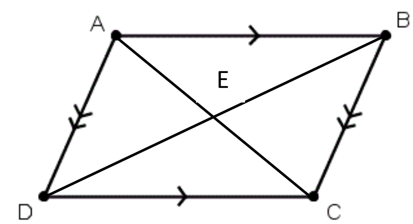
Write a proof of the parallelogram opposite angles theorem as it follows from the definition of parallelogram.



Hint: Draw diagonal $\overline{AC}$ to prove $\angle ABC \cong \angle ADC$. Then draw diagonal $\overline{BD}$ to prove that $\angle DAB \cong \angle DCB$.

5. **Parallelogram Diagonals Theorem**: If a quadrilateral is a parallelogram, then its diagonals bisect each other.

Fill in the blanks to complete the proof. Mark pairs of congruent sides and angles on the diagram.



Given: *ABCD* is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at *E*.
Prove: $\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$.

Since *ABCD* is a ________________, $\overline{AB} \parallel \overline{CD}$ by definition of ____________________.
It follows that $\angle BAC \cong$ _______ and $\angle ABD \cong$ __________ because Alternate Interior Angles formed by parallel lines are congruent.
Since opposite sides of parallelograms are congruent, $\overline{AB} \cong$ ________.
It follows that $\Delta AEB \cong \Delta$_________ by __________.

Thus, $\overline{AE} \cong \overline{EC}$ and __________________ by CPCTC. Therefore, the diagonals of *ABCD* bisect each other by definition of bisect.

Activity 3.5.5 Sufficient Conditions for Parallelograms

We know that if a quadrilateral has two pairs of parallel sides, then it is a parallelogram by definition. In this activity, you will explore what conditions are sufficient to prove that a quadrilateral has to be a parallelogram, without already knowing that both pairs of opposite sides are parallel.

1. Have each person in your group take one piece of linguine and break it so that you have two pairs of equal segments. Form a quadrilateral with your pieces. Depending on how you arrange the pieces you might get a kite, but you should be able to get a parallelogram. Was everyone in your group able to do that?

Now fill in the blanks.

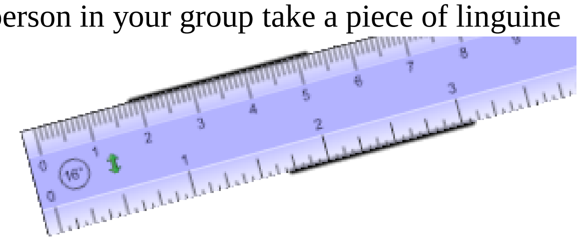
Parallelogram Opposite Sides Converse:

If the ___________________sides of a quadrilateral are _______________________, then the quadrilateral is a __________________________.

2. Have each person in your group take another piece of linguine and break into two unequal pieces. Let the pieces intersect at their midpoints but not be perpendicular. Either use your pencil or pieces of linguine to outline a quadrilateral around the diagonals. Did everyone in your group successfully make a parallelogram? Now, fill in the blanks.

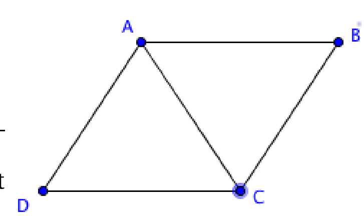
Parallelogram Diagonals Converse:
If the diagonals of a quadrilateral ______________ each other, then the quadrilateral is a _______________________.

3. What if you only know that one pair of opposite sides of a quadrilateral are congruent and parallel, but knew nothing about the other pair of sides? Is that sufficient evidence that the quadrilateral must be a parallelogram? Have each person in your group take a piece of linguine and break off two pieces of equal length. Place them somewhere on opposite sides of a ruler. Take the ruler away and connect the end points with two segments. Did you form a parallelogram? What about the rest of your group? Fill in the blanks.



Opposite Sides Congruent and Parallel Theorem: If one pair of opposite sides of a quadrilateral is ________________ and ________________, then the quadrilateral is a ______________________.

4. **Proof of Parallelogram Opposite Sides Converse**:



Given: In quadrilateral *ABCD*,
$\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$

Prove: *ABCD* is a parallelogram.

Start by drawing $\overline{AC}$.

In ΔABC and ΔADC, $\overline{AC}$ is congruent to ________ because it is a shared side.

We are also given that $\overline{AB} \cong \overline{CD}$ and _____________________.

We apply the _______ triangle congruence theorem to conclude that $\Delta ABC \cong$ ____________.

Since corresponding parts of congruent triangles are congruent, we can say that

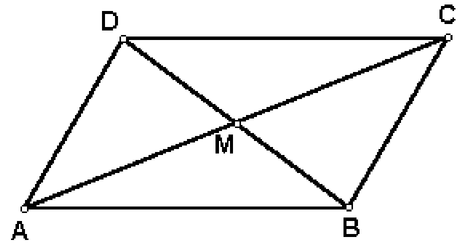
$\angle BCA \cong$ __________ which makes _____ || $\overline{BC}$ by the alternate interior angle converse.

We can also conclude that $\angle BAC \cong$ ______ by CPCTC.

Thus, _______ || _______ by the alternate interior angle converse.

Therefore, *ABCD* is a ________________ by the definition of _____________________.

5. **Proof of Parallelogram Diagonals Converse:**



Given: In quadrilateral *ABCD*,
diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other at M.

Prove: *ABCD* is a parallelogram.

Since, $\overline{AC}$ and $\overline{BD}$ bisect each other at M, then it follows that $\overline{AM} \cong$ _______ and $\overline{DM} \cong$ ______ by the definition of bisect.

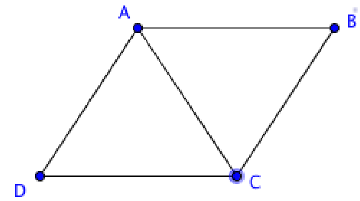
$\angle DMC$ and $\angle AMB$ are ______________ angles, so they must be congruent.

So, $\Delta DMC \cong$_____________ by the __________ triangle congruence theorem.

Then, $\overline{AB} \cong \overline{CD}$ by CPCTC. Complete the proof to show the other pair of opposite sides is congruent:

Since both pairs of opposite sides are congruent, the quadrilateral is a ______________________ by the Parallelogram Opposite Sides Converse.

6. **Proof of Opposite Sides Congruent and Parallel Theorem**



Given: In quadrilateral *ABCD*,
$\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$

Prove: *ABCD* is a parallelogram.

Start by drawing $\overline{AC}$.

Since $\overline{AB} \parallel \overline{CD}$, $\angle BAC \cong$ __________ by ______________________ theorem.

In ΔABC and ΔADC, $\overline{AC}$ is congruent to ________ since it is a shared side.

Since it is also given that $\overline{AB} \cong \overline{CD}$, we can conclude that $\Delta ABC \cong \Delta$ _____

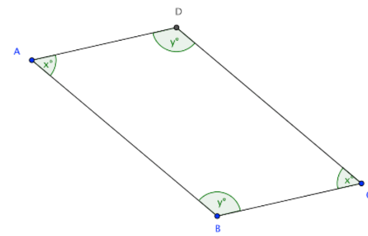
by _____________ triangle congruence.

It then follows that $\overline{BC} \cong \overline{AD}$ by ____________________.

Since both pairs of sides are congruent, and we just proved the ___________________ Converse,

ABCD must be a ________________________.

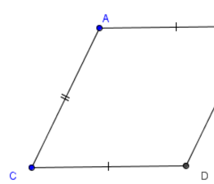
7. **If a quadrilateral has two pairs of opposite angles that are congruent, then the quadrilateral is a parallelogram.**

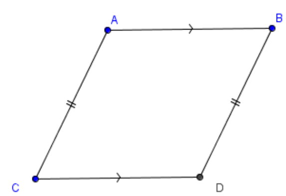


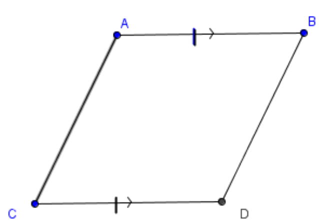
Use the diagram to the right to help prove that *ABCD* is a parallelogram.

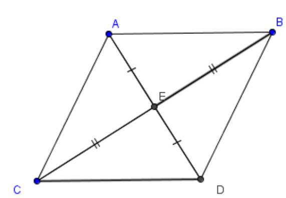
Statements	Reasons
$2x + 2y = 360$	
	Divide both sides of equation by 2.
∠ *A* and ∠ *B* are supplementary. Also ∠ *A* and ∠ *D* are supplementary.	
$\overline{AD} \| \overline{BC}$ and _____ || _____	Same Side Interior Angles Supplementary → Parallel Lines
	Definition of Parallelogram

8. Circle the quadrilaterals that have sufficient evidence to be a parallelogram, based on the markings on the figures.

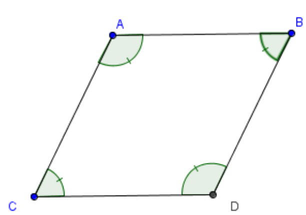
A) 

B) 

C) 



D)

E) 

Activity 3.5.6a Rectangles and Rhombuses

A rectangle is defined as an equiangular quadrilateral and a rhombus is defined as an equilateral quadrilateral. In this investigation, you will prove necessary and sufficient conditions for rectangles and rhombuses.

Review of Necessary Conditions of Rectangles and Rhombuses

1. If a quadrilateral is a rectangle, then the diagonals are ______________________________.

2. If a quadrilateral is a rhombus, then the diagonals are ______________________________.

Before we prove these conjectures, we need to prove that all rectangles are parallelograms and all rhombuses are parallelograms.

3. **Prove that *All Rectangles are Parallelograms***

Given: *ABCD* is a rectangle.
Prove: *ABCD* is a parallelogram.

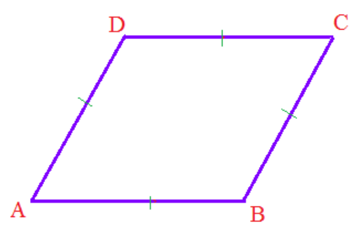
Since *ABCD* is a rectangle, $m\angle A = m\angle B = m\angle C = m\angle D$

This means that pairs of opposite angles are congruent: , $m\angle A =$ _____and $m\angle B =$ _____

In Activity 3.5.5 we proved that If a quadrilateral has two pairs of opposite angles that are ________________, then the quadrilateral is a parallelogram. Since *ABCD* has two pairs of opposite angles congruent, it must be a ______________________.

4. **Prove that *All Rhombuses are Parallelograms.***

Given: *ABCD* is a rhombus.
Prove: *ABCD* is a parallelogram.

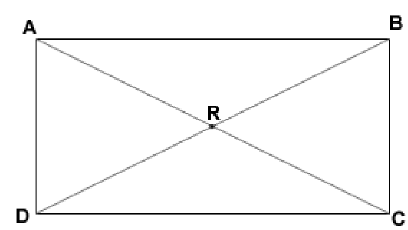


Since *ABCD* is a rhombus, *AB* = *BC* = *CD* = *DA*.

This means that pairs of opposite sides are congruent:; *AB* = ____ and *BC* = ______.

In Activity 3.5.5 we proved that If a quadrilateral has two pairs of opposite sides that are ________________, then the quadrilateral is a parallelogram. Since *ABCD* has two pairs of opposite sides congruent, it must be a ______________________.

5. Prove **Rectangle Diagonals Theorem:** If a parallelogram is a rectangle, then the diagonals are congruent.



Given: *ABCD* is a rectangle.
Prove: $\overline{AC} \cong \overline{BD}$

First, develop a plan for your proof by thinking backwards.

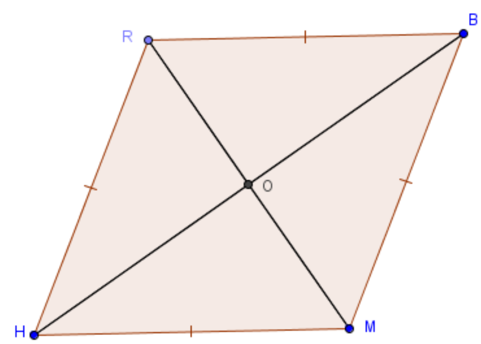
a) Name two triangles that you can prove are congruent that have the diagonals as corresponding parts. Hint: They may be overlapping triangles.

b) What three parts of those triangles can you prove are congruent?

Fill in the blanks in the proof below.

Since *ABCD* is a rectangle, opposite sides are ___________________. Therefore, ________≅_________. It is also true that ________≅_________ because it is a shared side. By definition of rectangle, ∠_________≅ ∠_________. By SAS, Δ___________≅ Δ___________. $\overline{AC} \cong \overline{BD}$ by ___________________.

6. Prove **Rhombus Diagonals Theorem:** If a parallelogram is a rhombus, then the diagonals are perpendicular.



Given: *RBMH* is a rhombus.
Prove: $\overline{RM} \perp \overline{BH}$

Since *ABCD* is a parallelogram and the diagonals of a parallelogram ___________ each other, *OH* = ____.

Also since the sides of a rhombus are congruent *HR* = *RB*.

OR = *OR* by the reflexive property.

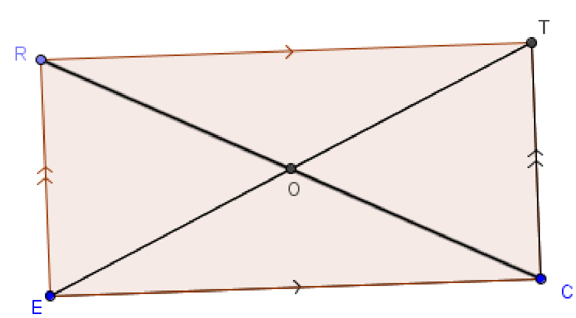
Therefore Δ*ROH* ≅ Δ*ROB* by the ________ Congruence Theorem.

∠_________≅ ∠_________ because corresponding parts of congruent triangles are congruent.

By the Linear Pair postulate: m∠_________ + m∠_________ = 180°.

Explain why we can now conclude that $\overline{RM}$ and $\overline{BH}$ are perpendicular:

7. **Prove Rectangle Diagonals Converse:** If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

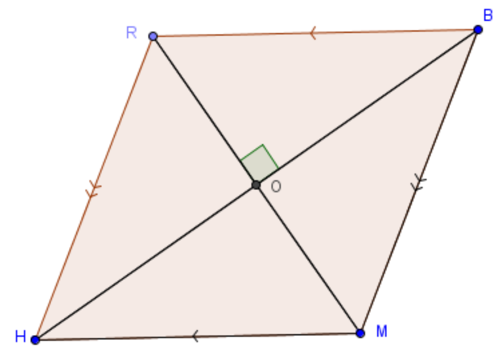


Given: *RTCE* is a parallelogram.
$\overline{RC} \cong \overline{TE}$

Prove: *RTCE* is a rectangle.

Since *RTCE* is a parallelogram, opposite sides are ________________, so $\overline{RE} \cong \overline{TC}$. $\overline{EC} \cong \overline{EC}$ because __________________. Since it is also given than ______ ≅ ______, by ___________ triangle congruence, it follows that Δ____________≅ Δ____________. By CPCTC, $\angle REC \cong$ _________. Since it has been proven that opposite angles of a parallelogram are ________________, then all four angles are ___________________. Then by definition of __________________, _______________________.

8. **Prove Rhombus Diagonals Converse:** If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.



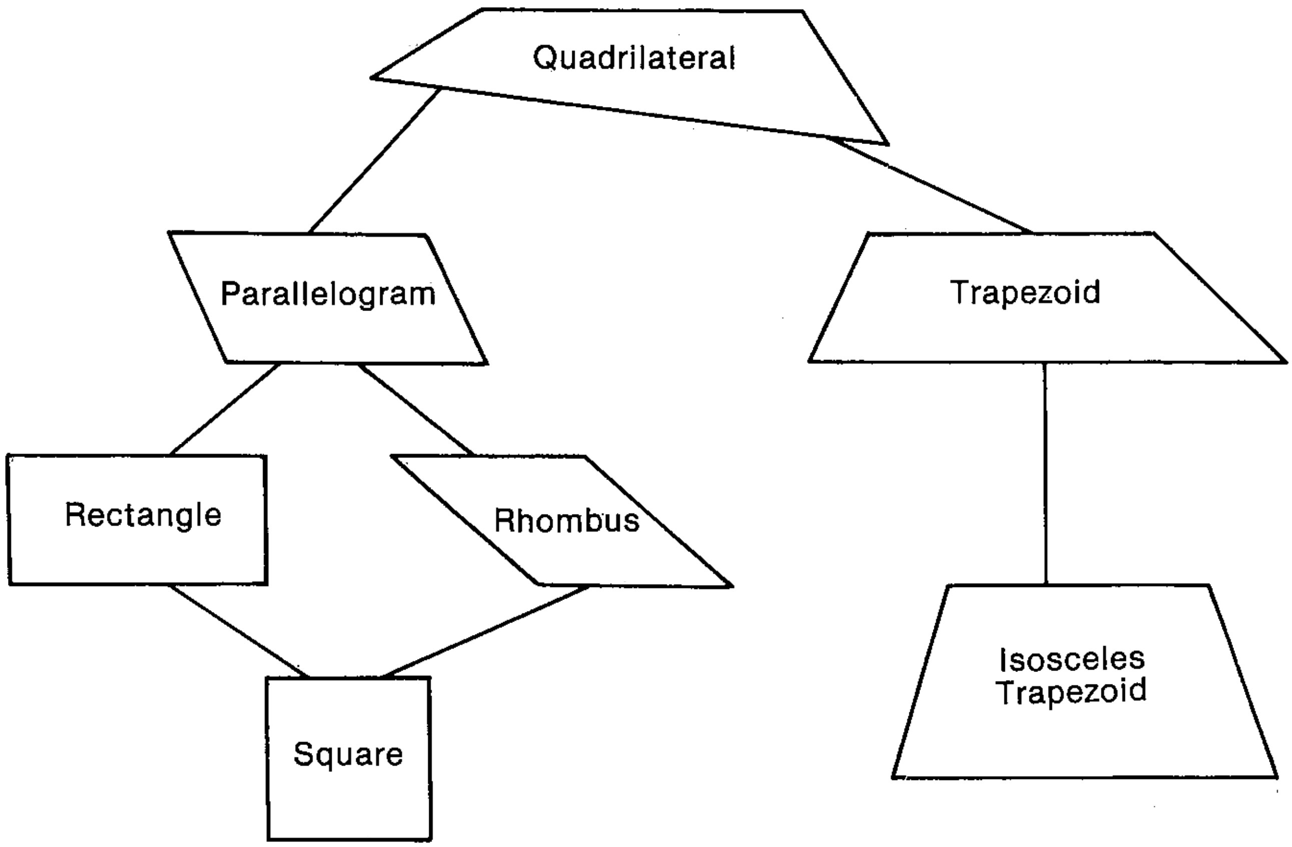
Given: *RBMH* is a parallelogram.
$\overline{RM} \perp \overline{BH}$
Prove: *RBMH* is a rhombus.

Plan: Use the parallelogram diagonals theorem to find congruent segments. Explain why all four small triangles are congruent in the figure. Then, explain how that proves the figure is a rhombus.

Write the proof.

3.5.6A

Special Quadrilaterals



CHARACTERISTICS	Parallel-ogram	Rect-angle	Rhom-bus	Square	Trapezoid	Isosceles Trapezoid
Both pairs of opposite sides parallel						
Diagonals congruent						
Both pairs of opposite sides congruent						
At least one right angle						
Both pairs of opposite angles congruent						
Exactly one pair of opposite sides parallel						
Diagonals perpendicular						
Consecutive sides congruent						
Consecutive angles congruent						
Diagonals bisect each other						
Diagonals bisect opposite angles						

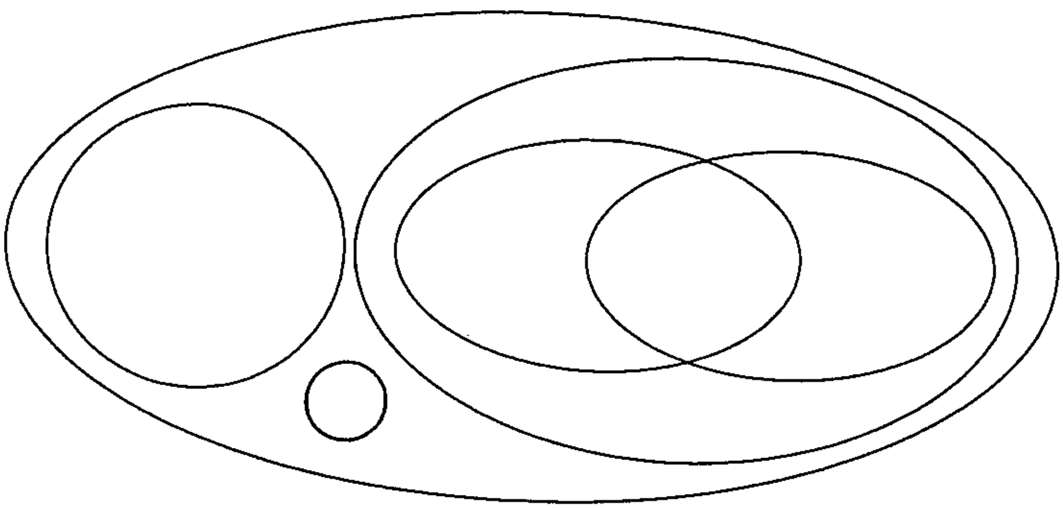
To answer these questions, you will need to find the definitions of these terms:

Quadrilateral
Parallelogram
Kite
Rectangle
Rhombus
Square
Trapezoid

Exercises:

1. What is a four sided figure with two pairs of parallel sides?
2. What is a four sided figure with exactly one pair of parallel sides?
3. What is a four sided figure with all sides congruent length?
4. What is a four sided figure with all angles congruent?
5. What is a four sided figure with all angles and sides congruent?
6. What is a four sided figure?
7. What is a four sided figure with opposite angles congruent?

8. Fill in each section of the Venn Diagram below with the 6 terms above.



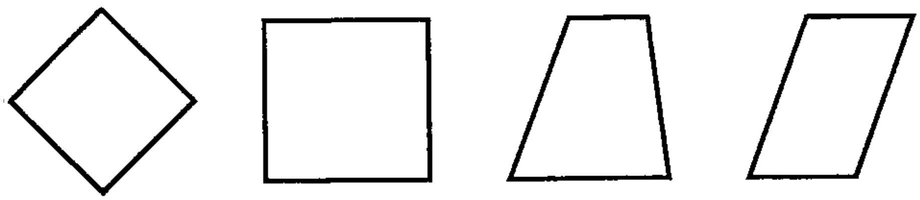
13. Copy and complete the table below by placing a yes (to mean always) or a no (to mean not always) in each empty space. Use what you know about special quadrilaterals.

	Kite	Isosceles trapezoid	Parallelogram	Rhombus	Rectangle
Opposite sides are parallel	No	No	Yes	Yes	Yes
Opposite sides are congruent	No	No	Yes	Yes	Yes
Opposite angles are congruent	No	No	Yes	Yes	Yes
Diagonals bisect each other	No	No	Yes	Yes	Yes
Diagonals are perpendicular	Yes	No	No	Yes	No
Diagonals are congruent	No	Yes	No	No	Yes
Exactly one line of symmetry	Yes	Yes	No	No	No
Exactly two lines of symmetry	No	No	No	Yes	Yes

Please write in complete sentences.

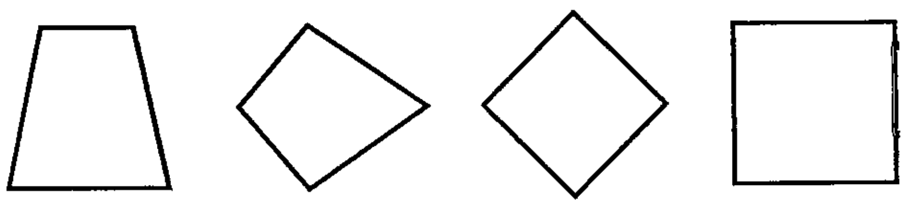
Which figure do you think does not belong in a set with the other three? Explain why it does not belong. There may be more than one possible answer.

1 •



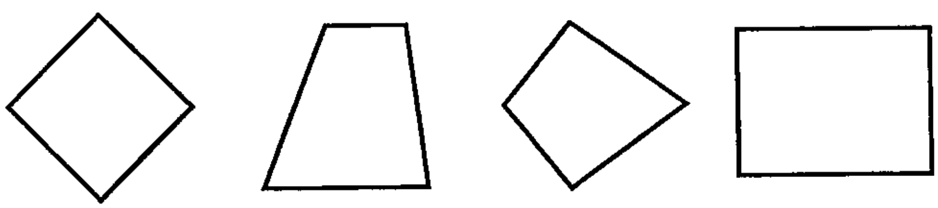
rhombus | square | trapezoid | parallelogram

2 •



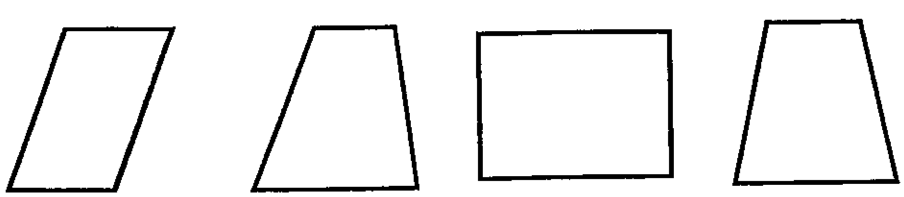
isosceles trapezoid | kite | rhombus | square

3 •



rhombus | trapezoid | kite | rectangle

4 •



parallelogram | trapezoid | rectangle | isosceles trapezoid

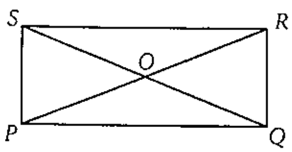
Lesson 5.6 • Properties of Special Parallelograms

1. *PQRS* is a rectangle and $OS = 16$.

$OQ =$ _____

$m\angle QRS =$ _____

$SQ =$ _____

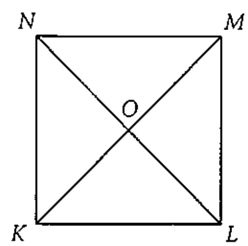


2. *KLMN* is a square and $NM = 8$.

$m\angle OKL =$ _____

$m\angle MOL =$ _____

Perimeter $KLMN =$ _____

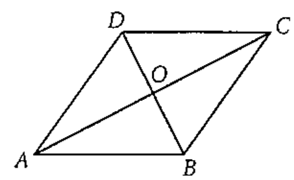


3. *ABCD* is a rhombus, $AD = 11$, and $DO = 6$.

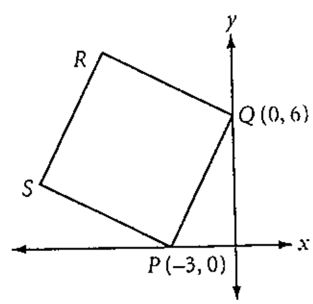
$OB =$ _____

$BC =$ _____

$m\angle AOD =$ _____



4. *PQRS* is a square. What are the coordinates of *R* and *S*?



In Exercises 5–13, match each description with all the terms that fit it.

a. Trapezoid
b. Isosceles triangle
c. Parallelogram
d. Rhombus
e. Kite
f. Rectangle
g. Square
h. All quadrilaterals

5. _____ Diagonals bisect each other.

6. _____ Diagonals are perpendicular.

7. _____ Diagonals are congruent.

8. _____ Measures of interior angles sum to 360°.

9. _____ Opposite sides are congruent.

10. _____ Opposite angles are congruent.

11. _____ Both diagonals bisect angles.

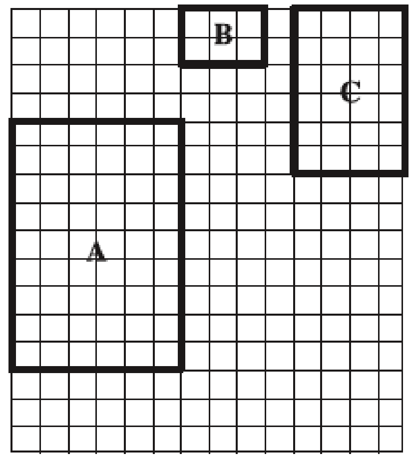
12. _____ Diagonals are perpendicular bisectors of each other.

13. _____ Has exactly one pair of congruent sides.

Activity 3.5.7 Areas of Quadrilaterals

The purpose of this activity is to derive the formulas for the area of a rectangle, parallelogram, triangle, trapezoid, and kite. Throughout this activity, the *base* may refer to any side of a polygon and the *height* is perpendicular to the base.

1. **Rectangle Area Postulate**
Assign each student in your group one of the rectangles at the right.



A: How many rows? _____ How many columns? _____
Area? _________

B: How many rows? _____ How many columns? _____
Area? _________

C: How many rows? _____ How many columns? _____
Area? _________

Rectangle Area Postulate - If *b* is the base and *h* is the height of a rectangle, then *Area* = _________________. Explain by referring to the rectangles above.

2. **Parallelogram Area Formula**
Go to http://people.wku.edu/tom.richmond/area.html and click parallelogram. Watch the animation and explain below what kind of transformations you saw. Does the base change? Does the height change? Does the area change? What is the area formula for the area of a parallelogram? Make a sketch of the animation.

Parallelogram Area Formula - If *b* is the base and *h* is the height of a parallelogram, then *Area* = _________________.

3. **Triangle Area Formula**
Use the same website and click triangle1. Watch the animation and explain below what kind of transformations you saw. Does the base change from the triangle to the parallelogram? Does the height change? Does the area change? What is the formula for the area of a triangle? Make a sketch of the animation.

Use the same website and click triangle2. Watch the animation and explain below what kind of transformations you saw. Does the base change from the triangle to the parallelogram? Does the height change? Does the area change? What is the formula for the area of a triangle? Make a sketch of the animation. Which animation do you like better?

Triangle Area Formula - If b is the base and h is the height of a triangle, then
Area = _________________.

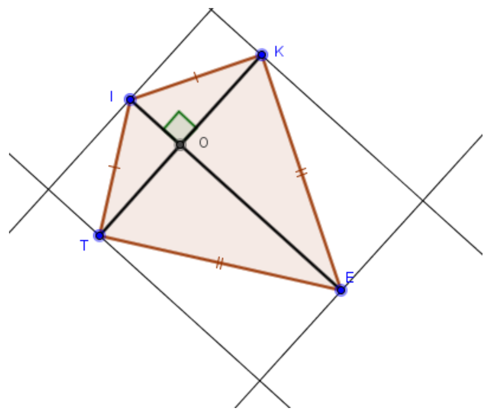
4. **Trapezoid Area Formula**
Use the same website and click trapezoid1. Watch the animation and explain below how the figure was divided. Show the algebraic steps in the animation. What is the formula for the area of a trapezoid? Make a sketch of the animation.

Use the same website and click trapezoid2. Watch the animation and explain below what kind of transformations you saw. Does the base change from the trapezoid to the parallelogram? Does the height change? Does the area change? What is the formula for the area of a trapezoid? Make a sketch of the animation. Which derivation do you like better for the trapezoid?

Trapezoid Area Formula – If b_1 and b_2 are the measures of the two bases of a trapezoid and h is the height, then *Area* = ____________________.

5. **Kite Area Formula**

For any kite, a rectangle can constructed around it such that the base and height of the rectangle are parallel to the diagonals of the kite. See picture to the right.



Label the diagonals d_1 and d_2. Then label the parallel sides of the rectangle d_1 and d_2. What is the area of the rectangle in terms of d_1 and d_2? How does the area of the kite compare to the area of the rectangle?

Kite Area Formula – If d_1 and d_2 are the diagonals of a kite, then *Area* = ___________________.

6. What formula(s) would you use to calculate the area of a rhombus? Explain your options.

Activity 3.6.1 Properties of Quadrilaterals on the Coordinate Plane

1. Review of important coordinate formulas and theorems:

   a. Write the slope formula.

   b. Write the midpoint formula.

   c. Write the distance formula.

   d. What must be true about the slopes of parallel lines?

   e. What must be true about the slopes of perpendicular lines?

2. What formulas or theorems from question 1 can help you prove the following quadrilaterals if you are given the ordered pairs of the four vertices of the quadrilateral.

   a. Trapezoid

   b. Parallelogram

   c. Rhombus

   d. Rectangle

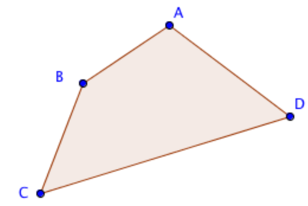
   e. Square

   f. Kite

3. Given the following slopes and distances, classify quadrilateral *ABCD*. Explain why you made your decision.

a.

Sides	Slopes	Distances
AB	1/3	3.16
BC	–1/4	4.12
CD	1/3	3.16
DA	–1/4	4.12



b.

Sides	Slopes	Distances
AB	3/6	6.71
BC	–2	2.24
CD	3/6	6.71
DA	–2	2.24

c.

Sides	Slopes	Distances
AB	1/4	8.25
BC	4	8.25
CD	1/4	8.25
DA	4	8.25

d.

Sides	Slopes	Distances
AB	3/4	5
BC	–4/3	5
CD	3/4	5
DA	–4/3	5

e.

	Slope	Distance
AB	0/4	4
BC	–1/2	2.236
CD	0/6	6
DA	1/2	2.236

f.

	Slope	Distance
AB	2/3	3.606
BC	–2/3	3.606
CD	4/5	6.403
DA	–4/5	6.403

4. For each of the following examples, calculate the length of all four sides and the slope of all four sides to most accurately classify each of the following quadrilaterals.

a. Quad *ADQU* with *A*(–2,3) , *D*(2,3), *Q*(3,–2), *U*(–1,–2)

b. Quad *PQRS* with *P*(–1,4) , *Q*(3,6), *R*(9,–3), *S*(5,–5)

c. Quad *QRSP* with *Q*(–4,6) , *R*(8,10), *S*(11,1), *P*(–1,–3)

d. Quad *EFGH* with *E*(0,–3) , *F*(–3,0), *G*(0,3), *H*(3,0)

e. Quad *DEFG* with *D*(–7,3) , *E*(–2,3), *F*(1,7), *G*(–4,7)

f. Quad *WXYZ* with *W*(0, 4), *X*(3, 6), *Y*(6, 4), *Z*(3, –1)

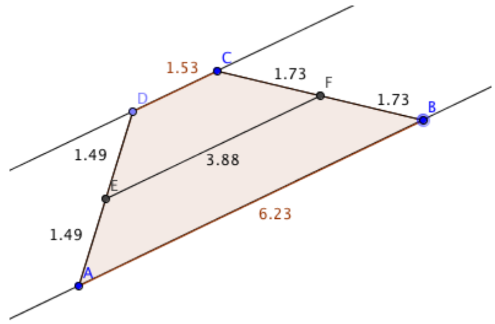
5. For the quadrilateral in 4b, prove that the diagonals bisect each other using one of the formulas from question 1. What theorem does this verify?

6. For the quadrilateral in 4c, prove that the diagonals are congruent using a formula from question 1. What theorem does this verify?

7. For the quadrilateral in 4d, prove that the diagonals are perpendicular bisectors of each other.

8. For the quadrilateral in 4e, prove that the diagonals are perpendicular. What theorem does this verify?

Activity 3.6.5 Midsegments



Open the file ctcoregeomACT365.ggb. The vertices of quadrilateral *ABCD* lie on two parallel lines as shown.

Move point *C* to demonstrate that DC is always parallel to $\overline{AB}$.

Move point *D* to show that *D* always lies on the line through *C* parallel to $\overline{AB}$. (However, don't move *D* beyond *C*—we'll do that later.)

1. Classify *ABCD* as a special quadrilateral. *ABCD* must be a ________________. Why?

2. The lengths of segment $\overline{ED}$, $\overline{EA}$, $\overline{FC}$, and $\overline{FB}$ are shown. As you move any of the vertices of the quadrilateral these quantities may change, but what always stays the same?

3. Therefore *E* is the __________ of $\overline{AD}$ and *F* is the ___________ of $\overline{BC}$.

4. In a trapezoid the two parallel sides are called **bases** and the other two sides are called **legs**. In *ABCD* the bases are _____ and ____. The legs are ______ and ______.

5. The **midsegment** of a trapezoid is the segment joining the midpoints of the legs. Name the midsegment of *ABCD*: _________

6. Compare the length of the midsegment of *ABCD* with the sum of the two bases. What do you notice? Does this relationship still hold when you move the vertices?

7. Find the slope of the midsegment of *ABCD* and compare it to the slopes of the bases. What do you notice? Does this relationship still hold when you move the vertices?

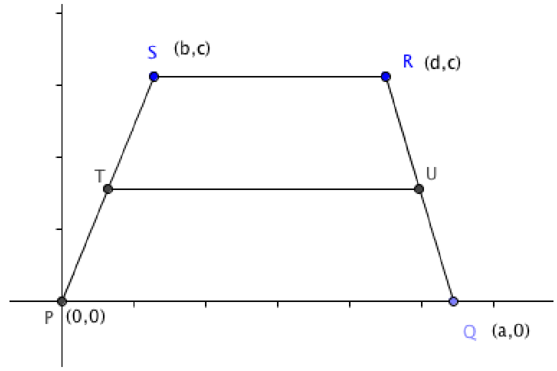
8. Based on your observations make a conjecture:

9. Now move *D* so that it coincides with *C*. What type of figure do you have now?

10. Continuing moving *D* so that it is on the other side of *C* from where it started. What happens? Is *ABCD* still a quadrilateral? Justify your answer.

Here are two theorems based on what we have observed:
Trapezoid Midsegment Theorem: The segment joining the two midpoints of the legs of a trapezoid is parallel to the bases and equal in length to the average of the lengths of the two bases.

Triangle Midsegment Theorem: The segment joining the midpoints of two sides of a triangle is parallel to the third side and equal in length to half the third side.



11-12. Prove the Trapezoid Midsegment Theorem. Recall that a trapezoid may be represented in the coordinate plane by $P(0,0)$, $Q(a, 0)$, $R(d, c)$ and $S(b, c)$. Let T be the midpoint of $\overline{PS}$ and U the midpoint of $\overline{QR}$.

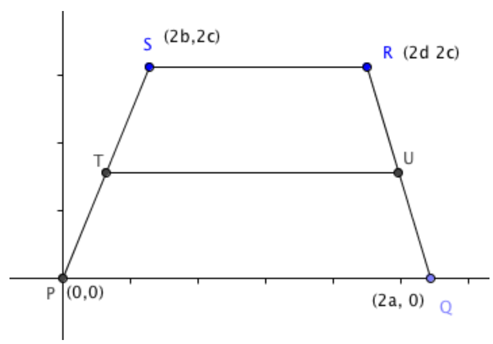
11. Start with the specific case where $a = 10$, $b = 4$, $c = 6$, and $d = 8$, that is the coordinates are $P(0,0)$, $Q(10, 0)$, $R(8,6)$ and $S(4,6)$.

a. Find coordinates of T(___,____) and U(_____, _____)

b. Find these lengths: PQ = _____ SR = _____ TU =______

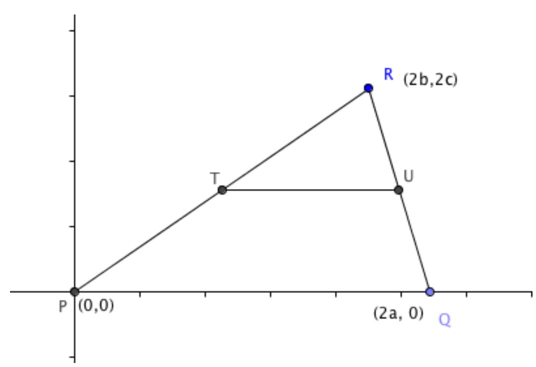
c. Find the slopes of $\overline{PQ}$, $\overline{SR}$, and $\overline{TU}$:

d. Now complete the proof.



12. Now prove the theorem in the general case. To make the calculations simpler we can double each variable coordinate so we have $P(0,0)$, $Q(2a, 0)$, $R(2d, 2c)$ and $S(2b, 2c)$.

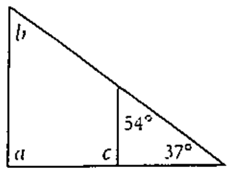
13. Prove the Triangle Midsegment Theorem. Use the coordinates shown in the figure at the right.



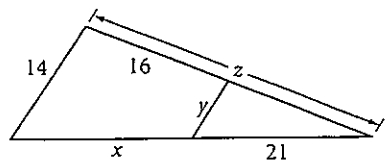
Lesson 5.4 • Properties of Midsegments

In Exercises 1–3, each figure shows a midsegment.

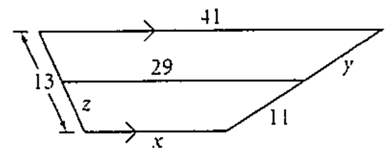
1. $a =$ ______, $b =$ ______, $c =$ ______



2. $x =$ ______, $y =$ ______, $z =$ ______



3. $x =$ ______, $y =$ ______, $z =$ ______

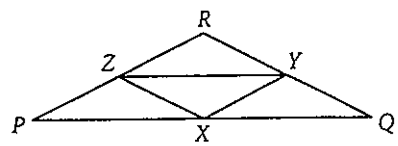


4. x, y, and z are midpoints. Perimeter $\triangle PQR = 132$, $RQ = 55$, and $PZ = 20$.

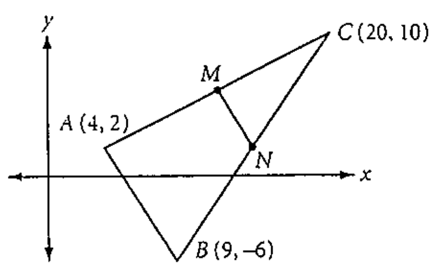
Perimeter $\triangle XYZ =$ ______

$PQ =$ ______

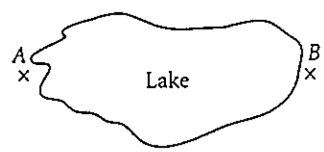
$ZX =$ ______



5. $\overline{MN}$ is the midsegment. Find the coordinates of M and N. Find the slopes of $\overline{AB}$ and $\overline{MN}$.



6. Explain how to find the width of the lake from A to B using a tape measure, but without using a boat or getting your feet wet.



Activity 3.6.6 Medians and Centroids

Recall the **Midpoint Formula** from Unit 1 Investigation 2:

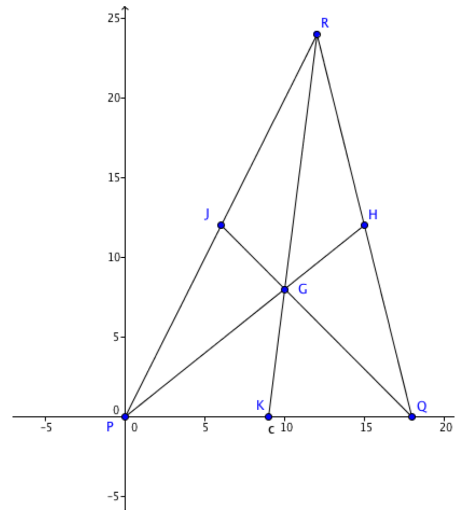
The coordinates of the midpoint of a line segment are the average of the coordinates of the two endpoints of the segment, that is

If the endpoints of the segment are (x_1, y_1) and (x_2, y_2), then the midpoint has coordinates $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

1. Plot ΔPQR on graph paper or using GeoGebra with $P(0,0)$, $Q(18,0)$, and $R(12,24)$.

2. Find the midpoint of each side

The midpoint of $\overline{QR}$ is H(______,______)
The midpoint of $\overline{RP}$ is J(______,______)
The midpoint of $\overline{PQ}$ is K(______,______)



3. Now join each vertex to the midpoint of the opposite side; that is, draw segments $\overline{PH}$, $\overline{QJ}$ and $\overline{RK}$. What do you notice?

4. $\overline{PH}$, $\overline{QJ}$ and $\overline{RK}$ are the **medians** of ΔPQR. In your own words, give a definition for "median of a triangle."

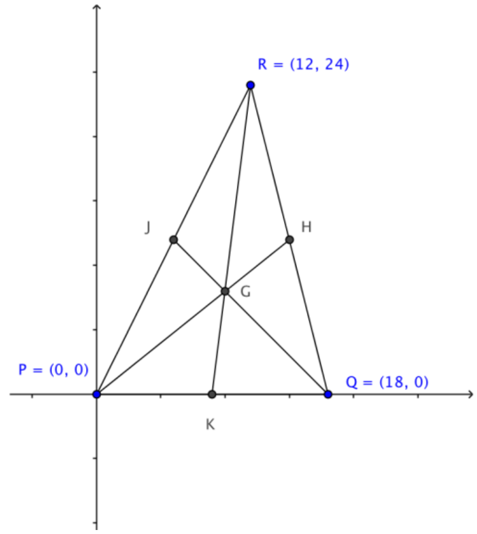
5. The **centroid** of a triangle may be described as the point whose coordinates are the average of the coordinates of the three vertices of the triangle. For ΔPQR find

the average of the x-coordinates of P, Q, and R: __________________

the average of the y-coordinates of P, Q, and R: __________________

The Coordinates of G the centroid of ΔPQR are G(_________, _________)

6. What connection do you see between the answers to questions 3 and 5?



7. We can show that *G* lies on each of the medians by finding the equations of the lines containing the medians.

Recall the **point-slope form** of the equation of a line: $y - y_1 = m(x - x_1)$

For each median, find the slope and the equation:

Slope of $\overleftrightarrow{PH}$ = ___________ Equation of $\overleftrightarrow{PH}$: ______________________________(1)

Slope of $\overleftrightarrow{QJ}$ = ___________ Equation of $\overleftrightarrow{QJ}$: ______________________________(2)

Slope of $\overleftrightarrow{RK}$ = ___________ Equation of $\overleftrightarrow{RK}$: ______________________________(3)

8. Show that the coordinates of *G* satisfy each of the three equations above:

Show that they satisfy equation (1):

Show that they satisfy equation (2):

Show that they satisfy equation (3):

9. You have shown that the coordinates of *G* satisfy each of the equations. Geometrically this means that the ______________ of a triangle is the point where the three ______________ intersect each other.

10. The centroid of a triangle is also known as the center of gravity. You can illustrate this property by drawing a triangle on heavy paper or card stock. Use a ruler to locate the midpoint of each side and draw the medians. Cut out the triangle and try balancing it on the tip of pencil at the centroid, as shown.

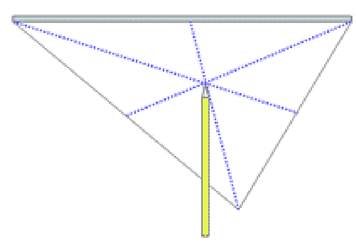
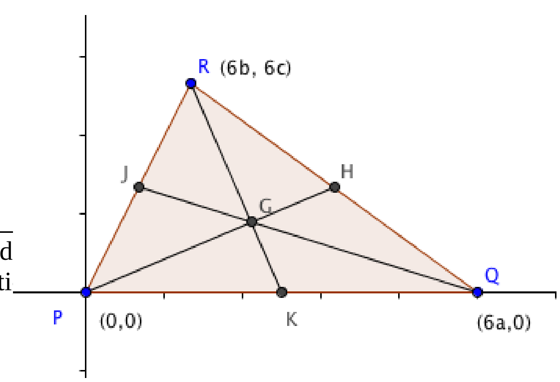


Figure from http://www.mathopenref.com/trianglecentroid.html

11. Prove the **Centroid Theorem**: The medians of a triangle intersect in one point. This point is the centroid of the triangle.

Fill in the blanks for this proof.



Let the vertices of the triangle be P(0,0), Q (6*a*, 0) and R(6*b*, 6*c*).

Then the coordinates of the centroid *G* are the averages of the coordinates of the vertices so we have *G*(______,_______).

The midpoints of the three sides are,

H(________, ________) for $\overline{QR}$,

J(________, ________) for $\overline{RP}$, and

K(________, ________) for $\overline{PQ}$.

We now find the equations of the medians:

Slope of $\overleftrightarrow{PH}$ = ___________ Equation of $\overleftrightarrow{PH}$: ______________________________(1)

Slope of $\overleftrightarrow{QJ}$ = ___________ Equation of $\overleftrightarrow{QJ}$: ______________________________ (2)

Slope of $\overleftrightarrow{RK}$ = ___________ Equation of $\overleftrightarrow{RK}$: ______________________________(3)

The coordinates of *G* into each of these equations to verify that *G* lies on all three lines:

Equation (1):

Equation (2):

Equation (3):

Activity 3.7.2 Regular Tessellations

A **regular tessellation** contains only regular polygons. All the polygons must have the same number of sides.

In this activity you will discover which regular polygons tile the plane and which do not.

You may cut out polygons from the template for this activity if you prefer.

1. Fill in this table:

Type of polygon	Number of sides	Measure of each interior angle	Will this regular polygon tile the plane? (yes/no)	If yes, how many polygons are there at each vertex?
Equilateral Triangle				
Square				
Regular Pentagon				
Regular Hexagon				
Regular Heptagon				
Regular Octagon				
Regular Nonagon				
Regular Decagon				

2. Explain why a regular polygon with more than 10 sides will not tile the plane.

3. Explain why a regular pentagon will not tile the plane.

4. Complete this sentence: The only regular polygons that tile the plane are ____________

____

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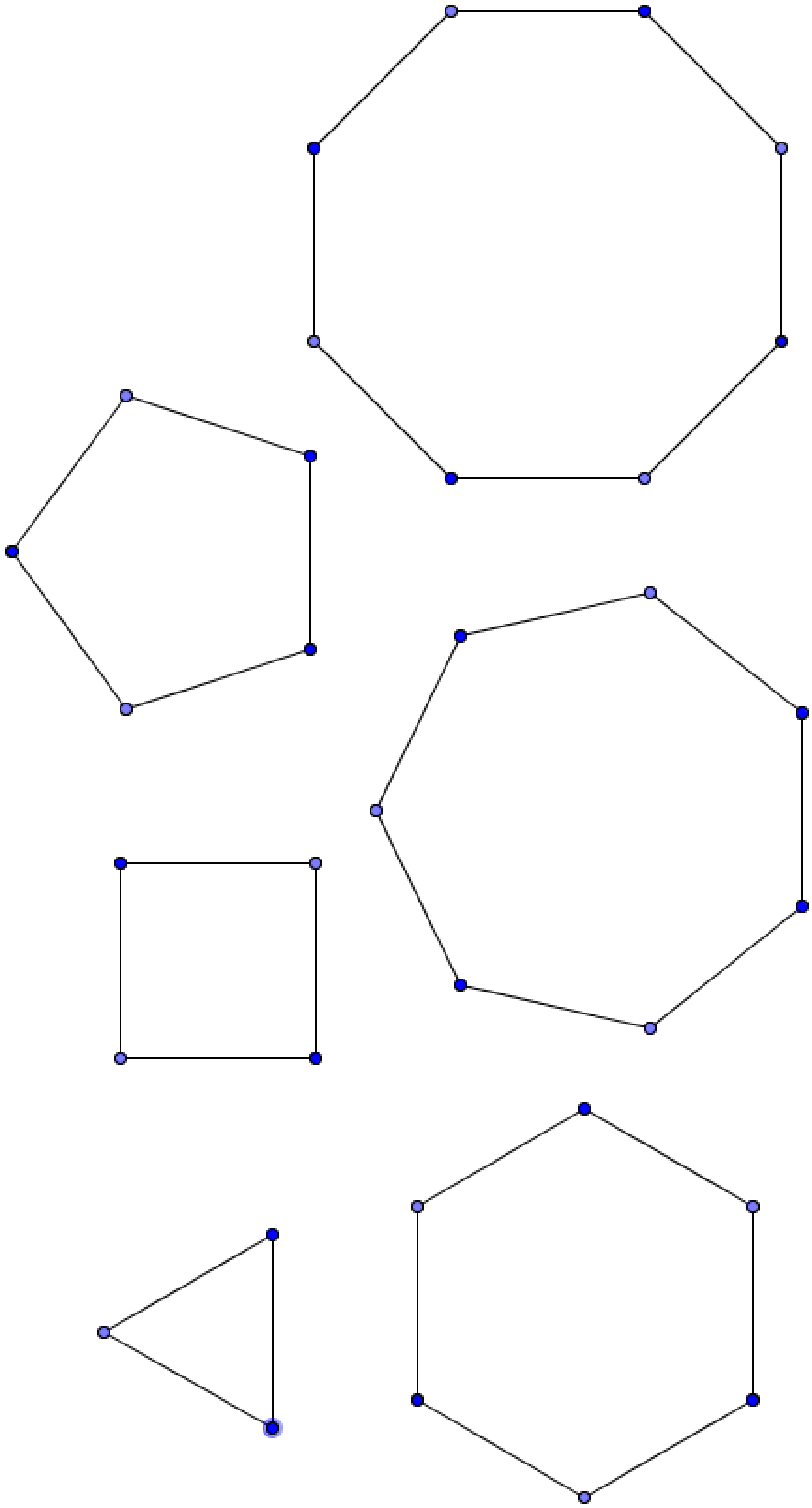

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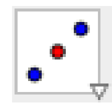
Activity 3.7.3 Tessellating Triangles

Show that any triangle may tile the plane.

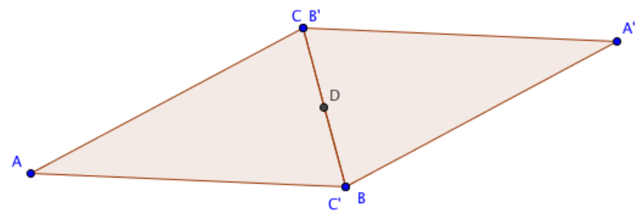
1. Pick three points and create ΔABC.

2. Find the midpoint of side $\overline{BC}$.
   (You may use Geogebra's shortcut with the midpoint or center tool.)



3. Rotate ΔABC by 180° around the midpoint.

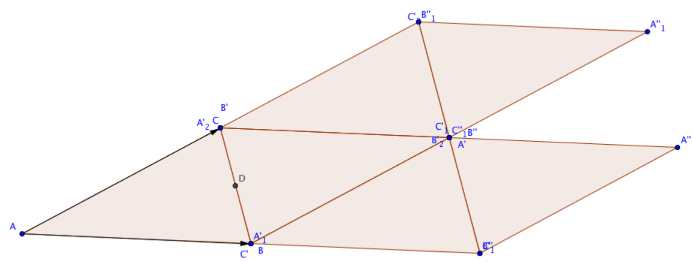
4. The two triangles now form what special quadrilateral? ________________________



5. Translate the two triangles with the vector from A to B.

6. Translate the same two triangles with the vector from A to C.

7. Keep translating until you have at least 12 copies of the original triangle.



8. Will you be able to continue filling the plane with copies of the triangle? Explain your reasoning.

9. How many triangles are there at each vertex?

10. What is the sum of the angles at each vertex?

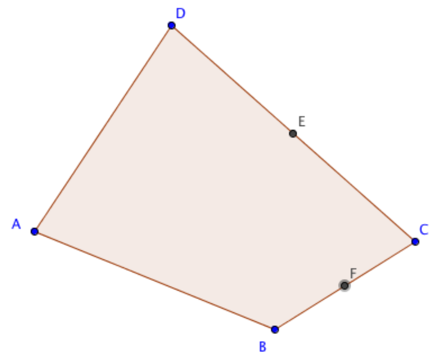
11. Now change the shape of ΔABC and observe what happens.

12. Which types of triangles will tile the plane?
    a. acute triangles
    b. right triangles
    c. obtuse triangles
    d. all of the above

    Chose a, b, c, or d and explain your choice.

Activity 3.7.4 Tessellating Quadrilaterals

Show that any quadrilateral may tile the plane.



1. Pick four points and create quadrilateral *ABCD*.

2. Place four copies of the quadrilateral at point *C*.

   Hint: Find the midpoints of $\overline{BC}$ and $\overline{CD}$ and rotate *ABCD* by 180° around these points.
   Then translate *ABCD* by the vector from *A* to *C* to fill in the remaining gap.

3. Show that you may now fill the plane by translating these four quadrilaterals as many times as you like. Keep translating until you have at least 20 copies of the original quadrilateral.

4. How many quadrilaterals are there at each vertex?

5. What is the sum of the angles at each vertex?

6. Now change the shape of quadrilateral *ABCD* and observe what happens.

7. Is it possible for a non-convex quadrilateral to tile the plane? Explain.